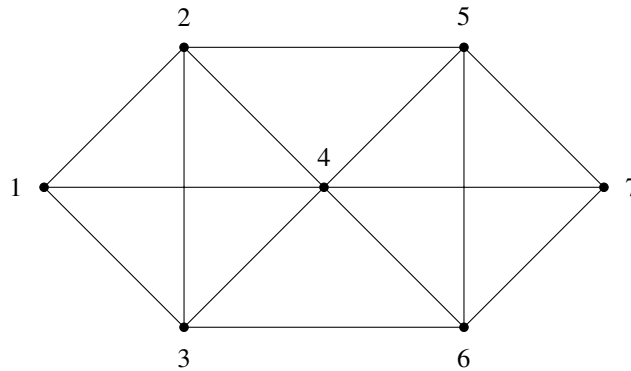


1 Eulerian Tour and Eulerian Walk

Note 5



- (a) Is there an Eulerian tour in the graph above? If no, give justification. If yes, provide an example.
- (b) Is there an Eulerian walk in the graph above? An Eulerian walk is a walk that uses each edge exactly once. If no, give justification. If yes, provide an example.
- (c) What is the condition that there is an Eulerian walk in an undirected graph? Briefly justify your answer.

2 Coloring Trees

Note 5

Prove that all trees with at least 2 vertices are *bipartite*: the vertices can be partitioned into two groups so that every edge goes between the two groups.

[*Hint*: Use induction on the number of vertices.]

3 Not everything is normal: Odd-Degree Vertices

Note 5

Claim: Let $G = (V, E)$ be an undirected graph. The number of vertices of G that have odd degree is even.

Prove the claim above using:

- (i) Direct proof (e.g., counting the number of edges in G). *Hint: in lecture, we proved that $\sum_{v \in V} \deg v = 2|E|$.*

(ii) Induction on $m = |E|$ (number of edges)

(iii) Induction on $n = |V|$ (number of vertices)