

## 1 Continuous Intro

Note 21

(a) Is

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

a valid density function? Why or why not? Is it a valid CDF? Why or why not?

(b) Calculate the PDF  $f_X(x)$ , along with  $\mathbb{E}[X]$  and  $\text{Var}(X)$  if the CDF of  $X$  is

$$F_X(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x}{\ell}, & 0 \leq x \leq \ell, \\ 1, & x \geq \ell \end{cases}$$

(c) Suppose  $X$  and  $Y$  are independent and have densities

$$f_X(x) = \begin{cases} 2x, & 0 \leq x \leq 1, \\ 0, & \text{otherwise,} \end{cases} \quad f_Y(y) = \begin{cases} 1, & 0 \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

What is their joint distribution? (Hint: for parts (c) and (d), we can use independence in much the same way that we did in discrete probability)

(d) Calculate  $\mathbb{E}[XY]$  for the  $X$  and  $Y$  in part (c).

## 2 Uniform Distribution

Note 21

You have two fidget spinners, each having a circumference of 10. You mark one point on each spinner as a needle and place each of them at the center of a circle with values in the range  $[0, 10)$  marked on the circumference. If you spin both (independently) and let  $X$  be the position of the first spinner's mark and  $Y$  be the position of the second spinner's mark, what is the probability that  $X \geq 5$ , given that  $Y \geq X$ ?

### 3 Darts Again

Note 21

Edward and Khalil are playing darts on a circular dartboard.

Edward's throws are uniformly distributed over the entire dartboard, which has a radius of 10 inches. Khalil has good aim; the distance of his throws from the center of the dartboard follows an exponential distribution with parameter  $\frac{1}{2}$ .

Say that Edward and Khalil both throw one dart at the dartboard. Let  $X$  be the distance of Edward's dart from the center, and  $Y$  be the distance of Khalil's dart from the center of the dartboard. What is  $\mathbb{P}[X < Y]$ , the probability that Edward's throw is closer to the center of the board than Khalil's? Leave your answer in terms of an unevaluated integral.

[*Hint:*  $X$  is not uniform over  $[0, 10]$ . Solve for the distribution of  $X$  by first computing the CDF of  $X$ ,  $\mathbb{P}[X < x]$ .]