

1 Exponential Distributions: Lightbulbs

Note 21

A brand new lightbulb has just been installed in our classroom, and you know the life span of a lightbulb is exponentially distributed with a mean of 50 days.

- (a) Suppose an electrician is scheduled to check on the lightbulb in 30 days and replace it if it is broken. What is the probability that the electrician will find the bulb broken?
- (b) Suppose the electrician finds the bulb broken and replaces it with a new one. What is the probability that the new bulb will last at least 30 days?
- (c) Suppose the electrician finds the bulb in working condition and leaves. What is the probability that the bulb will last at least another 30 days?

2 Interesting Gaussians

Note 21

- (a) If $X \sim N(0, \sigma_X^2)$ and $Y \sim N(0, \sigma_Y^2)$ are independent, then what is $\mathbb{E}[(X + Y)^k]$ for any odd $k \in \mathbb{N}$?

- (b) Let $f_{\mu,\sigma}(x)$ be the density of a $N(\mu, \sigma^2)$ random variable, and let X be distributed according to $\alpha f_{\mu_1, \sigma_1}(x) + (1 - \alpha) f_{\mu_2, \sigma_2}(x)$ for some $\alpha \in [0, 1]$. Compute $\mathbb{E}[X]$ and $\text{Var}(X)$. Is X normally distributed?

3 Condition on an Event

Note 21

The random variable X has the PDF

$$f_X(x) = \begin{cases} cx^{-2}, & \text{if } 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the value of c .

(b) Let A be the event $\{X > 1.5\}$. Calculate $\mathbb{P}[A]$ and the conditional PDF of X given that A has occurred.