CS 70	Discrete Mathematics and Probability Theory	
Spring 2023	Satish Rao and Babak Ayazifar	HW 00

Due: Saturday, 1/21, 4:00 PM
Grace period until Saturday, 1/21, 6:00 PM

Sundry

Before you start writing your final homework submission, state briefly how you worked on it. Who else did you work with? List names and email addresses. (In case of homework party, you can just describe the group.)

1 Administrivia

(a) Make sure you are on the course Ed (for Q&A) and Gradescope (for submitting homeworks, including this one). Find and familiarize yourself with the course website. What is its homepage's URL?

(b) Read the policies page on the course website.

  (i) What is the breakdown of how your grade is calculated?
  (ii) What is the attendance policy for discussions?
  (iii) When are homeworks released and when are they due?
  (iv) How many "drops" do you get for vitamins? For homework?
  (v) When is the midterm? When is the final?

2 Course Policies

Go to the course website and read the course policies carefully. Leave a followup on Ed if you have any questions. Are the following situations violations of course policy? Write "Yes" or "No", and a short explanation for each.

(a) Alice and Bob work on a problem in a study group. They write up a solution together and submit it, noting on their submissions that they wrote up their homework answers together.

(b) Carol goes to a homework party and listens to Dan describe his approach to a problem on the board, taking notes in the process. She writes up her homework submission from her notes, crediting Dan.

(c) Erin comes across a proof that is part of a homework problem while studying course material. She reads it and then, after she has understood it, writes her own solution using the same approach. She submits the homework with a citation to the website.

(d) Frank is having trouble with his homework and asks Grace for help. Grace lets Frank look at her written solution. Frank copies it onto his notebook and uses the copy to write and submit his homework, crediting Grace.

(e) Heidi has completed her homework using LaTeX. Her friend Irene has been working on a homework problem for hours, and asks Heidi for help. Heidi sends Irene her PDF solution, and Irene uses it to write her own solution with a citation to Heidi.

(f) Joe found homework solutions before they were officially released, and every time he got stuck, he looked at the solutions for a hint. He then cited the solutions as part of his submission.

3 Use of Ed

Ed is incredibly useful for Q&A in such a large-scale class. We will use Ed for all important announcements. You should check it frequently. We also highly encourage you to use Ed to ask questions and answer questions from your fellow students.

(a) Read the Ed Etiquette section of the course policies and explain what is wrong with the following hypothetical student question: "Can someone explain the proof of Theorem XYZ to me?" (Assume Theorem XYZ is a complicated concept.)

(b) When are the weekly posts released? Are they required reading?

(c) If you have a question or concern not directly related to the course content, where should you direct it?

4 Academic Integrity

Please write or type out the following pledge in print, and sign it.

> I pledge to uphold the university's honor code: to act with honesty, integrity, and respect for others, including their work. By signing, I ensure that all written homework I submit will be in my own words, that I will acknowledge any collaboration or help received, and that I will neither give nor receive help on any examinations.

5 Propositional Practice

Note 1

In parts (a)-(c), convert the English sentences into propositional logic. In parts (d)-(f), convert the propositions into English. In part (f), let $P(a)$ represent the proposition that a is prime.

(a) There is one and only one real solution to the equation $x^2 = 0$.

(b) Between any two distinct rational numbers, there is another rational number.

(c) If the square of an integer is greater than 4, that integer is greater than 2 or it is less than -2.

(d) $(\forall x \in \mathbb{R})\ (x \in \mathbb{C})$

(e) $(\forall x, y \in \mathbb{Z})(x^2 - y^2 \neq 10)$

(f) $(\forall x \in \mathbb{N})\ [\ (x > 1) \implies (\exists a, b \in \mathbb{N})\ ((a + b = 2x) \wedge P(a) \wedge P(b))\ ]$

6 Implication

Note 1

Which of the following assertions are true no matter what proposition Q represents? For any false assertion, state a counterexample (i.e. come up with a statement $Q(x,y)$ that would make the implication false). For any true assertion, give a brief explanation for why it is true.

(a) $\exists x \exists y Q(x,y) \implies \exists y \exists x Q(x,y)$.

(b) $\forall x \exists y Q(x,y) \implies \exists y \forall x Q(x,y)$.

(c) $\exists x \forall y Q(x,y) \implies \forall y \exists x Q(x,y)$.

(d) $\exists x \exists y Q(x,y) \implies \forall y \exists x Q(x,y)$.

7 Equivalences with Quantifiers

Note 1

Evaluate whether the expressions on the left and right sides are equivalent in each part, and briefly justify your answers.

(a) $\forall x \exists y\, (P(x) \implies Q(x,y)) \stackrel{?}{\equiv} \forall x\, (P(x) \implies \exists y\, Q(x,y))$

(b) $\forall x\, ((\exists y\, Q(x,y)) \implies P(x)) \stackrel{?}{\equiv} \forall x \exists y\, (Q(x,y) \implies P(x))$

(c) $\neg \exists x \forall y\, (P(x,y) \implies \neg Q(x,y)) \stackrel{?}{\equiv} \forall x\, ((\exists y\, P(x,y)) \wedge (\exists y\, Q(x,y)))$

8 Preserving Set Operations

Note 0
Note 2

For a function f, define the image of a set X to be the set $f(X) = \{y \mid y = f(x) \text{ for some } x \in X\}$. Define the inverse image or preimage of a set Y to be the set $f^{-1}(Y) = \{x \mid f(x) \in Y\}$. Prove the following statements, in which A and B are sets. By doing so, you will show that inverse images preserve set operations, but images typically do not.

Recall: For sets X and Y, $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$. To prove that $X \subseteq Y$, it is sufficient to show that $(\forall x)\ ((x \in X) \implies (x \in Y))$.

(a) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.

(b) $f^{-1}(A \setminus B) = f^{-1}(A) \setminus f^{-1}(B)$.

(c) $f(A \cap B) \subseteq f(A) \cap f(B)$, and give an example where equality does not hold.

(d) $f(A \setminus B) \supseteq f(A) \setminus f(B)$, and give an example where equality does not hold.