Due: Saturday, 4/8, 4:00 PM
Grace period until Saturday, 4/8, 6:00 PM

Sundry

Before you start writing your final homework submission, state briefly how you worked on it. Who else did you work with? List names and email addresses. (In case of homework party, you can just describe the group.)

1 Maybe Lossy Maybe Not

Note 15

Let us say that Alice would like to send a message to Bob, over some channel. Alice has a message of length 6.

(a) Packets are dropped with probability p. If Alice sends 7 packets, what is probability that Bob can successfully reconstruct Alice's message using polynomial interpolation?

(b) Again, packets can be dropped with probability p. The channel may additionally corrupt 1 packet after deleting packets. Alice realizes this and sends 10 packets for a message of length 6. What is the probability that Bob receives enough packets to successfully reconstruct Alice's message using Berlekamp-Welch?

(c) Again, packets can be dropped with probability p. This time, packets may be corrupted with probability q. A packet being dropped is independent of whether or not is corrupted (i.e. a packet may be both corrupted and dropped). Consider the original scenario where Alice sends 7 packets for a message of length 6. What is probability that Bob can correctly reconstruct Alice's message using polynomial interpolation on all of the points he receives?

2 Class Enrollment

Note 15
Note 19

Lydia has just started her CalCentral enrollment appointment. She needs to register for a geography class and a history class. There are no waitlists, and she can attempt to enroll once per day in either class or both. The CalCentral enrollment system is strange and picky, so the probability of enrolling successfully in the geography class on each attempt is p_g and the probability of enrolling successfully in the history class on each attempt is p_h. Also, these events are independent.

(a) Suppose Lydia begins by attempting to enroll in the geography class everyday and gets enrolled in it on day G. What is the distribution of G?

(b) Suppose she is not enrolled in the geography class after attempting each day for the first 7 days. What is $\mathbb{P}[G = i \mid G > 7]$, the conditional distribution of G given $G > 7$?

(c) Once she is enrolled in the geography class, she starts attempting to enroll in the history class from day $G+1$ and gets enrolled in it on day H. Find the expected number of days it takes Lydia to enroll in both the classes, i.e. $\mathbb{E}[H]$.

Suppose instead of attempting one by one, Lydia decides to attempt enrolling in both the classes from day 1. Let G be the number of days it takes to enroll in the geography class, and H be the number of days it takes to enroll in the history class.

(d) What is the distribution of G and H now? Are they independent?

(e) Let A denote the day she gets enrolled in her first class and let B denote the day she gets enrolled in both the classes. What is the distribution of A?

(f) What is the expected number of days it takes Lydia to enroll in both classes now, i.e. $\mathbb{E}[B]$?

(g) What is the expected number of classes she will be enrolled in by the end of 30 days?

3 Swaps and Cycles

Note 15
Note 16

We'll say that a permutation $\pi = (\pi(1), \ldots, \pi(n))$ contains a *swap* if there exist $i, j \in \{1, \ldots, n\}$ so that $\pi(i) = j$ and $\pi(j) = i$, where $i \neq j$.

(a) What is the expected number of swaps in a random permutation?

(b) What about the variance?

(c) In the same spirit as above, we'll say that π contains a *k-cycle* if there exist $i_1, \ldots, i_k \in \{1, \ldots, n\}$ with $\pi(i_1) = i_2, \pi(i_2) = i_3, \ldots, \pi(i_k) = i_1$. Compute the expectation of the number of k-cycles.

4 Throwing Frisbees

Note 15
Note 16

Shahzar and his $n-1$ friends stand in a circle and play the following game: Shahzar throws a frisbee to one of the other people in the circle randomly, with each person being equally likely, and thereafter, the person holding the frisbee throws it to someone else in the circle, again uniformly at random. The game ends when someone throws the frisbee back to Shahzar.

(a) What is the expected number of times the frisbee is thrown through the course of the game?

(b) What is the expected number of people that never get the frisbee during the game?

5 Balls and Bins

Note 16

Throw n balls into m bins, where m and n are positive integers. Let X be the number of bins with exactly one ball. Compute $\mathrm{Var}(X)$. Your final answer should not contain any summations.

6 Will I Get My Package?

Note 15
Note 16

A delivery guy in some company is out delivering n packages to n customers, where n is a natural number greater than 1. Not only does he hand each customer a package uniformly at random from the remaining packages, he opens the package before delivering it with probability $1/2$. Let X be the number of customers who receive their own packages unopened.

(a) Compute the expectation $\mathbb{E}[X]$.

(b) Compute the variance $\mathrm{Var}(X)$.