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Solving for $Q(x)$ and $E(x)$...

For all points $1, \dots, i, n+2k = m$,

$$Q(i) = R(i)E(i) \pmod{p}$$

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$$\begin{aligned} a_3 + a_2 + a_1 + a_0 &\equiv 3(1 - b_0) \pmod{7} \\ a_3 + 4a_2 + 2a_1 + a_0 &\equiv 1(2 - b_0) \pmod{7} \end{aligned}$$

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$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

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$Q(x) = x^3 + 6x^2 + 6x + 5$.

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```
             -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
```

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$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2
          -----------------
x - 2 ) x^3   + 6 x^2 + 6 x + 5
        x^3   - 2 x^2
```

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```
                  1 x^2
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                  1 x^2 + 6 x + 5
```

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```
                  1 x^2 + 1 x
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
```

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```
                 1 x^2 + 1 x
           -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
               1 x^2 + 6 x + 5
               1 x^2 - 2 x
               ---------------
                         x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x + 1
        -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
          x^3  - 2 x^2
          ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
```

Example: finishing up.

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$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
         -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
               1 x^2 + 6 x + 5
               1 x^2 - 2 x
               ---------------
                         x + 5
                         x - 2
                         -----
                             0
```

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```
                1 x^2 + 1 x + 1
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
                          -----
                              0
```

$P(x) = x^2 + x + 1$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

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What is $\frac{x-2}{x-2}$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
          -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
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What is $\frac{x-2}{x-2}$? 1

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```
                  1 x^2 + 1 x + 1
         -----------------
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         x^3  - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
          ------------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$? Hole there?

Error Correction: Berlekamp-Welsh

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$.

Check your undersanding.

You have error locator polynomial!

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.
See where it is 0.

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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Example: dealing with $\frac{x-2}{x-2}$ at $x = 2$.

Yaay!!

Berlekamp-Welsh algorithm decodes correctly when k errors!

Poll

Say you sent a message of length 4, encoded as $P(x)$ where one sends packets $P(1),...P(8)$.

You recieve packets $R(1),...R(8)$.

Packets 1 and 4 are corrupted.

(A) $R(1) \neq P(1)$
(B) The degree of $P(x)E(x) = 3 + 2 = 5$.
(C) The degree of $E(x)$ is 2.
(D) The number of coefficients of $P(x)$ is 4.
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(A), (C), (E). (F) doesn't type check!

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Communicate n packets, with k erasures.

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Reed-Solomon codes.

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  - Polynomial division! $P(x) = Q(x)/E(x)$!

Reed-Solomon codes. Welsh-Berlekamp Decoding.

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

How many packets? $n+2k$
Why?
k changes to make diff. messages overlap
How to encode? With polynomial, $P(x)$. Of degree? $n-1$.
Recover?
Reconstruct error polynomial, $E(X)$, and $P(x)$!
Nonlinear equations.
Reconstruct $E(x)$ and $Q(x) = E(x)P(x)$. Linear Equations.
Polynomial division! $P(x) = Q(x)/E(x)$!

Reed-Solomon codes. Welsh-Berlekamp Decoding. Perfection!

Cool.

Really Cool!

Poll: How big is infinity?

Poll: How big is infinity?

Mark what's true.

(A) There are more real numbers than natural numbers.
(B) There are more rational numbers than natural numbers.
(C) There are more integers than natural numbers.
(D) pairs of natural numbers $>>$ natural numbers.

Same Size. Poll.

Two sets are the same size?

Same Size. Poll.

Two sets are the same size?

(A) Bijection between the sets.
(B) Count the objects and get the same number. same size.
(C) Counting to infinity is hard.

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(A), (B).
(C)?

Countable.

How to count?

Countable.

How to count?

0,

Countable.

How to count?

0, 1,

Countable.

How to count?

0, 1, 2,

Countable.

How to count?

0, 1, 2, 3,

Countable.

How to count?

$0, 1, 2, 3, \ldots$

Countable.

How to count?

0, 1, 2, 3, ...

The Counting numbers.

Countable.

How to count?

0, 1, 2, 3, . . .

The Counting numbers.
The natural numbers! N

Countable.

How to count?

0, 1, 2, 3, ...

The Counting numbers.
The natural numbers! N

Definition: S is **countable** if there is a bijection between S and some subset of N.

Countable.

How to count?

0, 1, 2, 3, $\ldots$

The Counting numbers.
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If the subset of N is finite, S has finite **cardinality**.

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The Counting numbers.
The natural numbers! N

Definition: S is **countable** if there is a bijection between S and some subset of N.

If the subset of N is finite, S has finite **cardinality**.

If the subset of N is infinite, S is **countably infinite**.

Countably infinite subsets.

Enumerating a set implies countable.
Corollary: Any subset T of a countable set S is countable.

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Enumerate T as follows:

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All countably infinite sets have the same cardinality.

Enumeration example.

All binary strings.

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$B = \{0, 1\}^*$.

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$B = \{\phi; , 0, 00, 000, 0000, ...\}$
Never get to 1.

More fractions?

Enumerate the rational numbers in order...

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$0, \ldots, 1/2, ..$

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Can't list in "order".

Pairs of natural numbers.

Consider pairs of natural numbers: $N \times N$

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E.g.: $(1,2)$, $(100,30)$, etc.

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For finite sets S_1 and S_2,
then $S_1 \times S_2$

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Pairs of natural numbers.

Enumerate in list:

Pairs of natural numbers.

Enumerate in list:
$(0,0)$,

Pairs of natural numbers.

Enumerate in list:
$(0,0),(1,0),$

Pairs of natural numbers.

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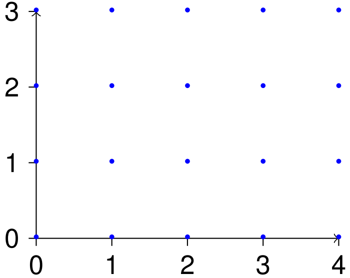

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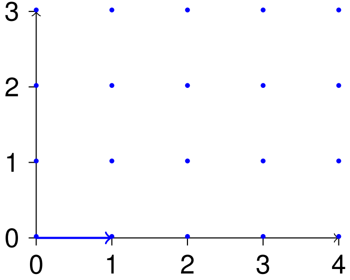

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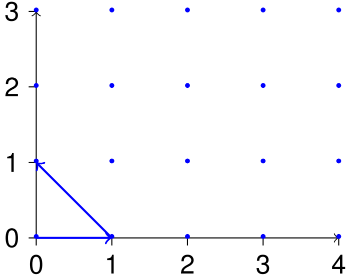

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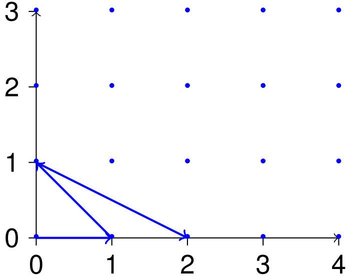

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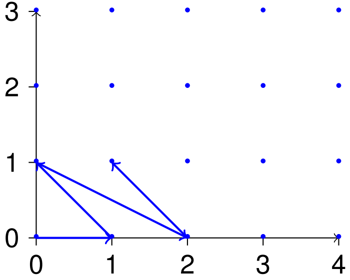

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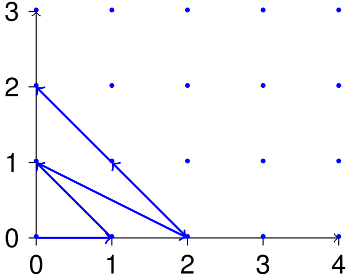

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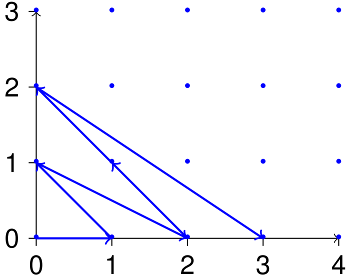

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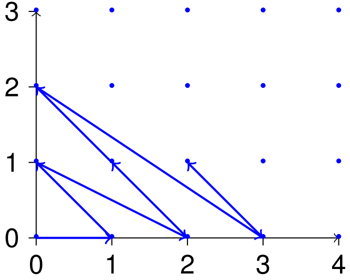

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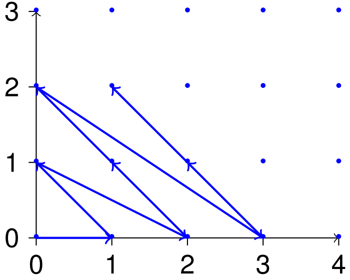

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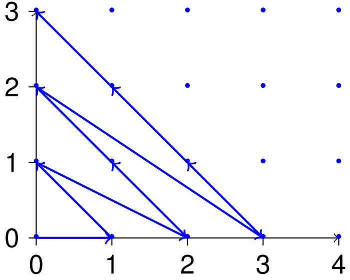

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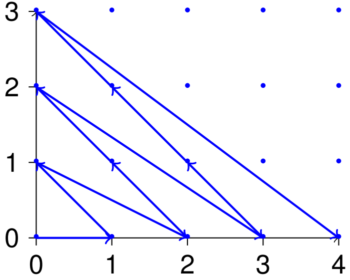

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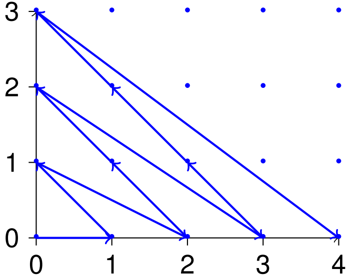

The pair (a,b), is in first $\approx (a+b+1)(a+b)/2$ elements of list!

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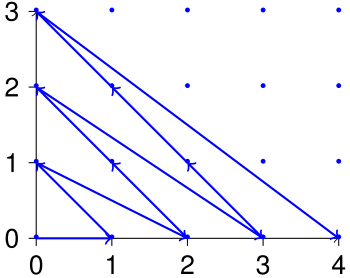

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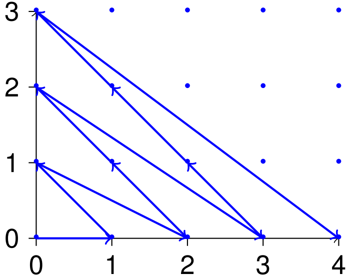

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Countably infinite.

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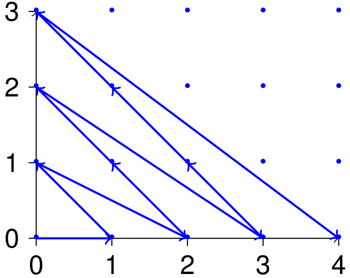

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Countably infinite.

Same size as the natural numbers!!

Poll.

Enumeration to get bijection with naturals?

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Enumeration to get bijection with naturals?

(A) Integers: First all negatives, then positives.
(B) Integers: By absolute value, break ties however.
(C) Pairs of naturals: by sum of values, break ties however.
(D) Pairs of naturals: by value of first element.
(E) Pairs of integers: by sum of values, break ties.
(F) Pairs of integers: by sum of absolute values, break ties.

Poll.

Enumeration to get bijection with naturals?

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(B),(C), (F).

Rationals?

Positive rational number.

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Lowest terms: a/b

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$a, b \in N$

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Negative rationals are countable.

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Put all rational numbers in a list.

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All rational numbers?

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Put all rational numbers in a list.

First negative, then nonegative ??? No!

Repeatedly and alternatively take one from each list.

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  Interleave Streams in 61A

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The rationals are countably infinite.

Real numbers..

Real numbers are same size as integers?

The reals.

Are the set of reals countable?

The reals.

Are the set of reals countable?

Lets consider the reals $[0, 1]$.

The reals.

Are the set of reals countable?

Lets consider the reals $[0, 1]$.

Each real has a decimal representation.

The reals.

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.500000000...

The reals.

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The reals.

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Lets consider the reals $[0,1]$.

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Diagonalization.

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Construct "diagonal" number: .77

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Construct "diagonal" number: .7767

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If reals are countable then so is $[0, 1]$.

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1. Assume that a set S can be enumerated.

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6. Contradiction.

Another diagonalization.

The set of all subsets of N.

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Example subsets of N: $\{0\}$,

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There is a listing, L, that contains all subsets of N.

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Theorem: The set of all subsets of N is not countable.
(The set of all subsets of S, is the **powerset** of N.)

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Mark parts of proof.

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(A) Integers are larger than naturals cuz obviously.
(B) Integers are countable cuz, interleaving bijection.
(C) Reals are uncountable cuz obviously!
(D) Reals can't be in a list: diagonal number not on list.
(E) Powerset in list: diagonal set not in list.

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(B), (C)?, (D), (E)

The Continuum hypothesis.

There is no set with cardinality between the naturals and the reals.

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First of Hilbert's problems!

Cardinalities of uncountable sets?

Cardinality of $[0, 1]$ smaller than all the reals?

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$$f(x) = \begin{cases} x + \frac{1}{2} & 0 \leq x \leq 1/2 \\ \frac{1}{4x} & x > 1/2 \end{cases}$$

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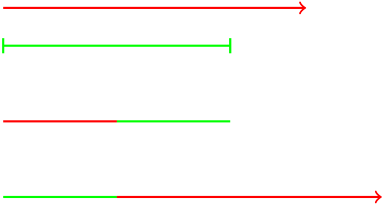

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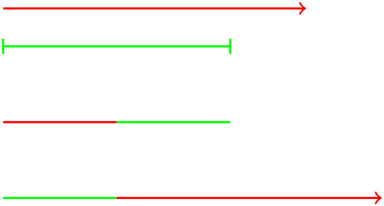

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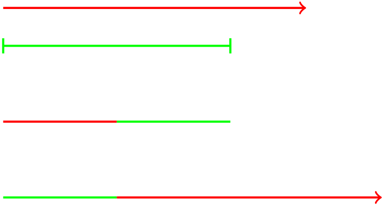

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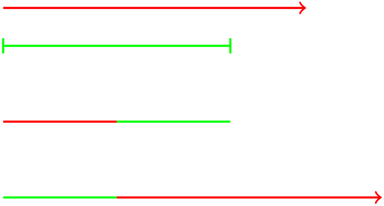

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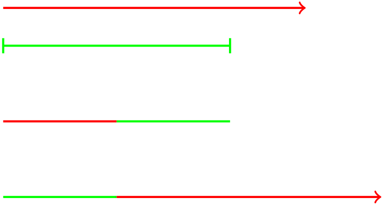

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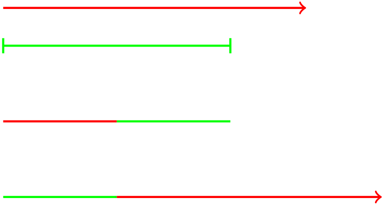

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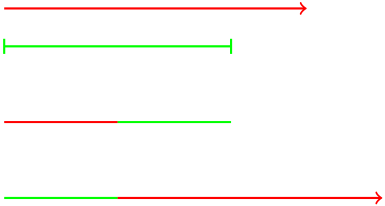

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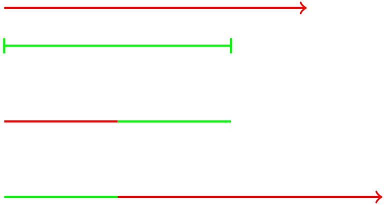
Bijection!

Cardinalities of uncountable sets?

Cardinality of $[0,1]$ smaller than all the reals?

$f : R^+ \rightarrow [0,1]$.

$$f(x) = \begin{cases} x + \frac{1}{2} & 0 \leq x \leq 1/2 \\ \frac{1}{4x} & x > 1/2 \end{cases}$$



One to one. $x \neq y$
If both in $[0,1/2]$, a shift $\implies f(x) \neq f(y)$.
If neither in $[0,1/2]$ a division $\implies f(x) \neq f(y)$.
If one is in $[0,1/2]$ and one isn't, different ranges $\implies f(x) \neq f(y)$.
Bijection!

$[0,1]$ is same cardinality as nonnegative reals!

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There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

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There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

The powerset of a set is the set of all subsets.

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Its all true. It's all a problem.

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Recall: powerset of the naturals is not countable.

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See Logicomix by Doxiaidis, Papadimitriou (was professor here), Papadatos, Di Donna.