The Barber!

The barber shaves all and only people who do not shave themselves.

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Self reference.

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Can a program refer to a program?

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Uh oh....

Is it actually useful?

Write me a program checker!

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Check that the compiler works!

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How about.. Check that the compiler terminates on a certain input.

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$HALT(P, I)$

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Text string can be an input to a program.

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Run P on I and check!

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How long do you wait?

Implementing HALT.

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Determines if $P(I)$ (P run on I) halts or loops forever.

Run P on I and check!

How long do you wait?

Something about infinity here, maybe?

Halt does not exist.

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$HALT(P, I)$

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Theorem: There is no program HALT.

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What is he talking about?

Yes! No!...

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(A) He is confused.
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(C) Welch-Berlekamp
(D) Professor is just strange.

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(B) and (D) maybe? and maybe (A).

Professor does me some love Welch-Berlekamp though!

Halt and Turing.

Proof:

Halt and Turing.

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Turing(P)

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Contradiction. Program HALT does not exist!

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Questions?

Another view of proof: diagonalization.

Any program is a fixed length string.

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Fixed length strings are enumerable.

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Program halts or not on any input, which is a string.

Another view of proof: diagonalization.

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Program halts or not on any input, which is a string.

	P_1	P_2	P_3	$\cdots$
P_1	H	H	L	$\cdots$
P_2	L	L	H	$\cdots$
P_3	L	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

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and is different from every P_i on the diagonal.

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Turing is not on list.

Another view of proof: diagonalization.

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Turing is not on list. Turing is not a program.

Another view of proof: diagonalization.

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Turing can be constructed from Halt.
Halt does not exist!

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	P_1	P_2	P_3	$\cdots$
P_1	H	H	L	$\cdots$
P_2	L	L	H	$\cdots$
P_3	L	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

Halt - diagonal.
Turing - is not Halt.
and is different from every P_i on the diagonal.
Turing is not on list. Turing is not a program.
Turing can be constructed from Halt.
Halt does not exist! □

Programs?

What are programs?

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(A) Instructions.
(B) Text.
(C) Binary String.
(D) They run on computers.

Programs?

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(A) Instructions.
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All are correct.

Proof play by play.

Assumed HALT(P, I) existed.

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What is P?

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$\Longrightarrow$ HALT is not a program.

Questions?

We are so smart!

Wow, that was easy!

We are so smart!

Wow, that was easy!

We should be famous!

No computers for Turing!

In Turing’s time.

No computers for Turing!

In Turing's time.

No computers.

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No computers.

Adding machines.

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Concept of program as data wasn't really there.

Turing machine.

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– an (infinite) tape with characters

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Turing: AI, self modifying code, learning...

Turing and computing.

Just a mathematician?

Turing and computing.

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“Wrote” a chess program.

Turing and computing.

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Simulated the program by hand to play chess.

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The polish machine...the *bomba.*

Computing on top of computing...

Computer, assembly code, programming language, browser, html, javascript..

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Computer, assembly code, programming language, browser, html, javascript..

We can't get enough of building more Turing machines.

Undecidable problems.

Does a program, P, print "Hello World"?

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Turing: personal.

Tragic ending...

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  (A bite from the apple....) accident?
- British Government apologized (2009) and pardoned (2013).

Back to technical..

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Every person who doesn't shave themselves is shaved by the barber.

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...Text of Halt...

Back to technical..

This statement is a lie. Neither true nor false!

Every person who doesn't shave themselves is shaved by the barber.

Who shaves the barber?

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Program is text, so we can pass it to itself,

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Program is text, so we can pass it to itself,
or refer to self.

Summary: decidability.

Computer Programs are an interesting thing.

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Like Math.

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Computer Programs cannot completely "understand" computer programs.

Computation is a lens for other action in the world.

Probability

What’s to come?

Probability

What's to come? Probability.

Probability

What's to come? Probability.

A bag contains:

Probability

What's to come? Probability.

A bag contains:

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

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For now:

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What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

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For now: Counting!

Probability

What's to come? Probability.

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For now: Counting!

Later: Probability.

The future in this course.

What's to come?

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Count blue. Count total. Divide. **Chances?**

The future in this course.

What's to come? Probability.

A bag contains:

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Count blue. Count total. Divide. **Chances?**

(A) Red Probability is $3/8$
(B) Blue probability is $3/9$
(C) Yellow Probability is $2/8$
(D) Blue probability is $3/8$

The future in this course.

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Today:

The future in this course.

What's to come? Probability.

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Count blue. Count total. Divide. **Chances?**

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Today: Counting!

Outline: basics

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn't matter.

Probability is soon..but first let’s count.

Count?

How many outcomes possible for k coin tosses?
How many poker hands?
How many handshakes for n people?
How many diagonals in a n sided convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?
How many ways can I divide up 5 dollars among 3 people?

Using a tree..

How many 3-bit strings?

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How many different sequences of three bits from $\{0, 1\}$?

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How would you make one sequence?

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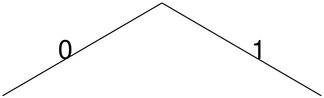

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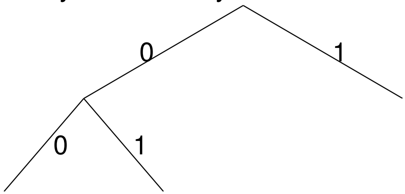

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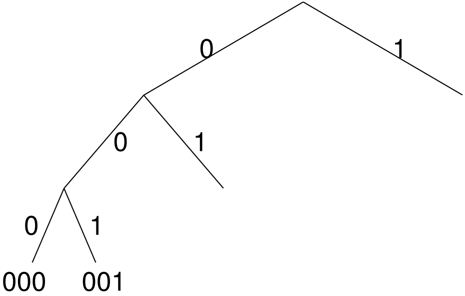

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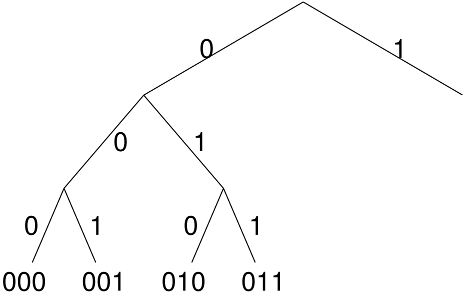

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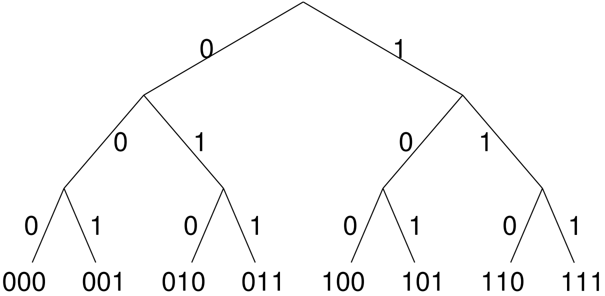

8 leaves which is $2 \times 2 \times 2$.

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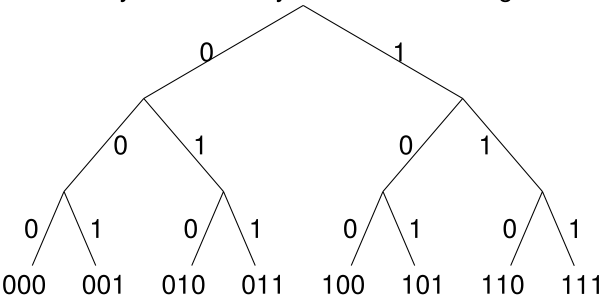

8 leaves which is $2 \times 2 \times 2$. One leaf for each string.

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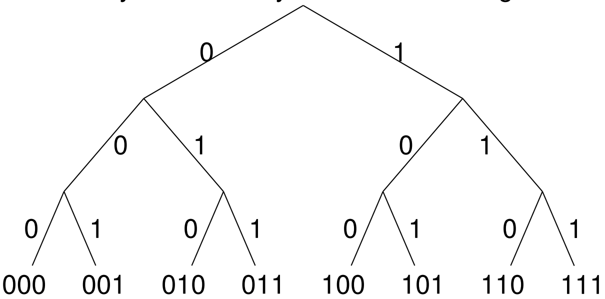

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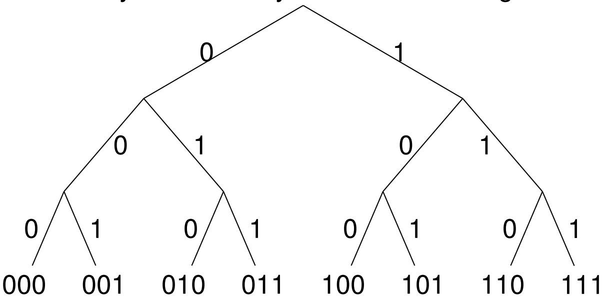

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8 3-bit strings!

First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, …, then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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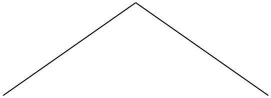

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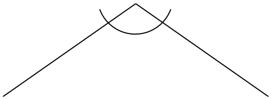

n_1

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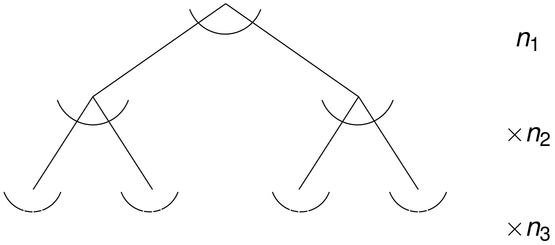

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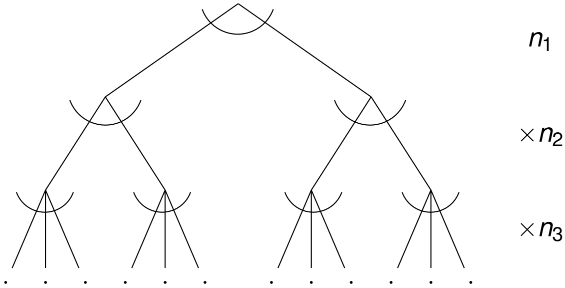

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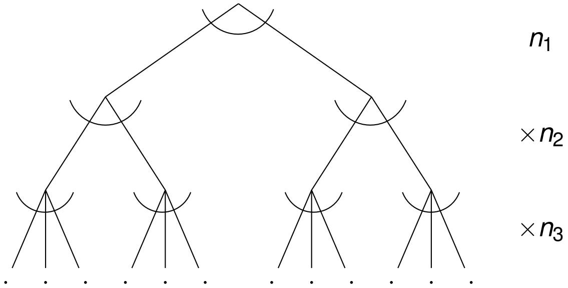

In picture, $2 \times 2 \times 3 = 12$!

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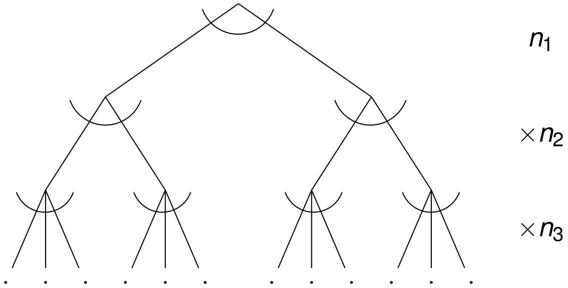

In picture, $2 \times 2 \times 3 = 12$!

Poll

Mark whats corect.

Poll

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(A) $|\text{10 digit numbers}| = 10^{10}$
(B) $|k \text{ coin tosses}| = 2^k$
(C) $|\text{10 digit numbers}| = 9 * 10^9$
(D) $|n \text{ digit base } m \text{ numbers}| = m^n$
(E) $|n \text{ digit base } m \text{ numbers}| = (m-1)m^{n-1}$

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(A) or (C)? (D) or (E)? (B) are correct.

Using the first rule..

How many outcomes possible for k coin tosses?

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How many outcomes possible for k coin tosses?

2 ways for first choice,

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

2×2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

Using the first rule..

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Using the first rule..

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$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
10

Using the first rule..

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$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
$10 \times$

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How many 10 digit numbers?

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$10 \times 10 \cdots$

Using the first rule..

How many outcomes possible for k coin tosses?

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$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^k$

Using the first rule..

How many outcomes possible for *k* coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^k$

How many *n* digit base *m* numbers?

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^k$

How many n digit base m numbers?

m ways for first,

Using the first rule..

How many outcomes possible for k coin tosses?

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m ways for first, m ways for second, ...

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How many n digit base m numbers?

m ways for first, m ways for second, ...
m^n

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$10 \times 10 \cdots \times 10 = 10^k$

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m ways for first, m ways for second, ...
m^n

(Is 09, a two digit number?)

Using the first rule..

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$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

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$10 \times 10 \cdots \times 10 = 10^k$

How many n digit base m numbers?

m ways for first, m ways for second, ...
m^n

(Is 09, a two digit number?)

If no. Then $(m-1)m^{n-1}$.

Functions, polynomials.

How many functions f mapping S to T?

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$|T|$ ways to choose for $f(s_1)$,

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How many polynomials of degree d modulo p?

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Functions, polynomials.

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How many polynomials of degree d modulo p?

p ways to choose for first coefficient, p ways for second, ...

Functions, polynomials.

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How many polynomials of degree d modulo p?

p ways to choose for first coefficient, p ways for second, ...
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Functions, polynomials.

How many functions f mapping S to T?

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Questions?

Permutations.

[1] By definition: $0! = 1$.

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How many 10 digit numbers **without repeating a digit**?

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So total number is $|S| \times |S|-1 \cdots 1 = |S|!$
A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

[2]When each unordered object corresponds equal numbers of ordered objects.

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$52 \times 51 \times 50 \times 49 \times 48$

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Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

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(The "!" means factorial, not Exclamation.)

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Number of orderings for a poker hand: "5!"

Can write as...

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

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$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

---

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Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

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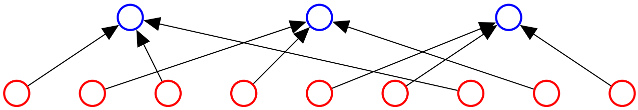

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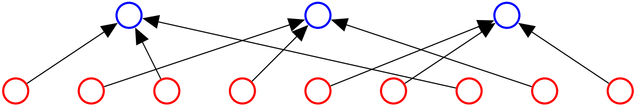

How many red nodes (ordered objects)?

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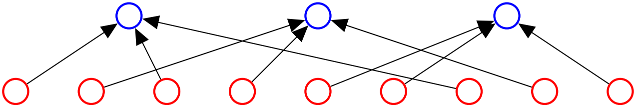

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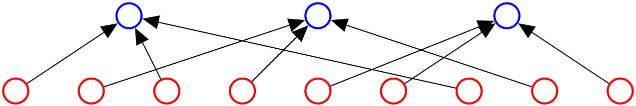

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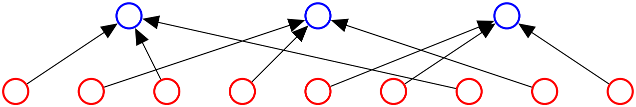

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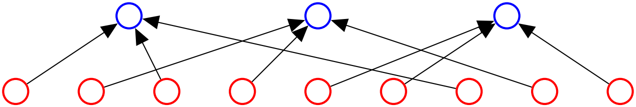

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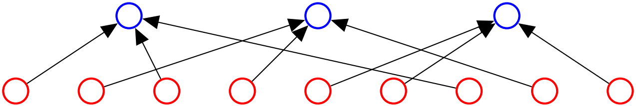

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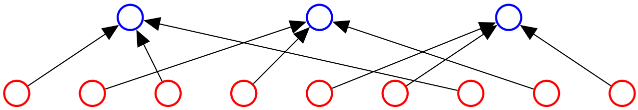

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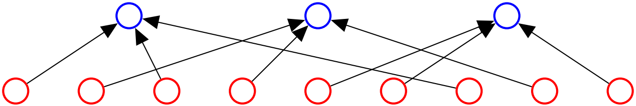

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How many poker deals?

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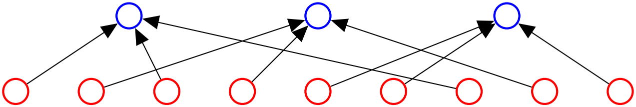

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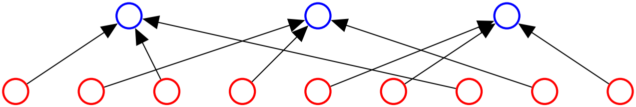

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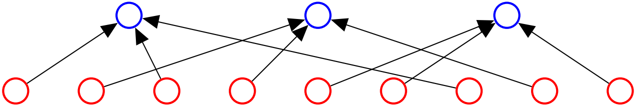

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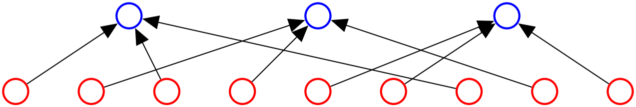

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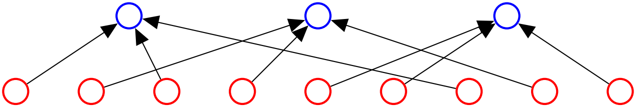

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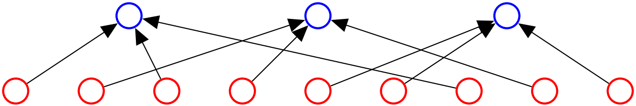

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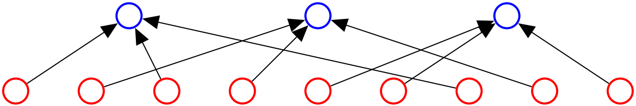

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Choose 2 out of n?

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$$\frac{n!}{(n-k)!}$$

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Choose k **out of** n?

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Notation: $\binom{n}{k}$ **and pronounced "**n **choose** k**."**

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Familiar? Questions?

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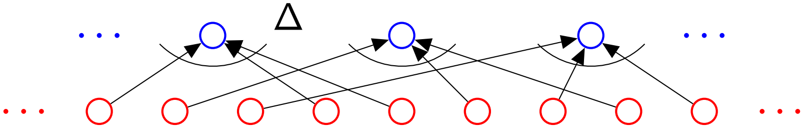
Second rule: when order doesn't matter divide...



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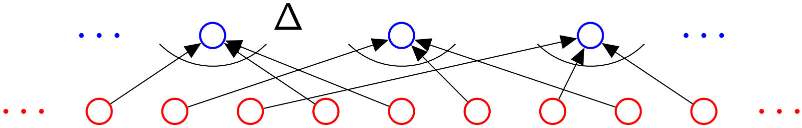

3 card Poker deals: 52

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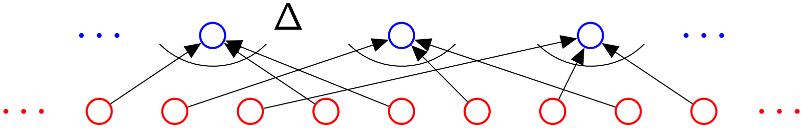

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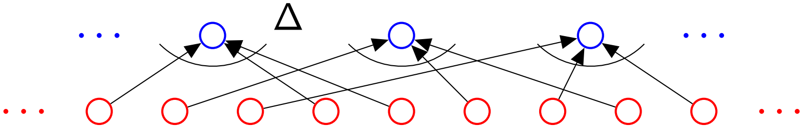

3 card Poker deals: $52 \times 51 \times 50$

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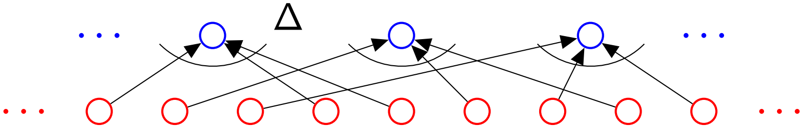

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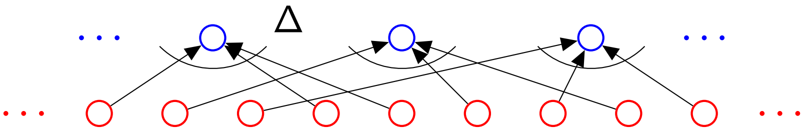

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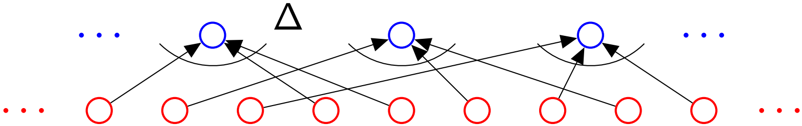

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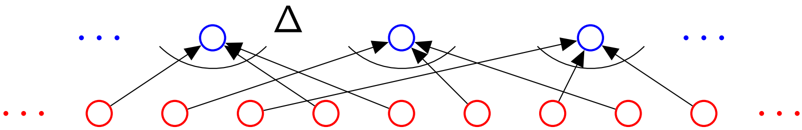

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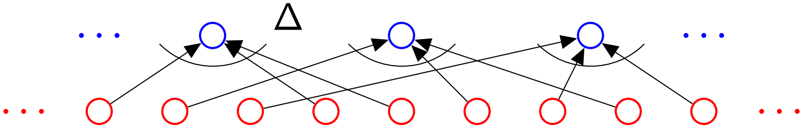

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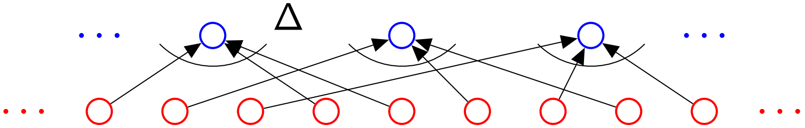

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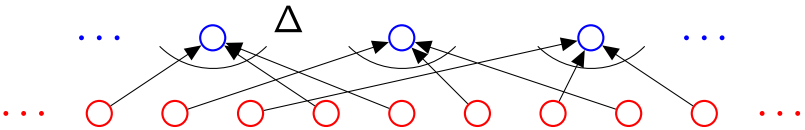

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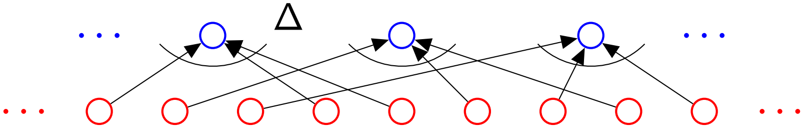

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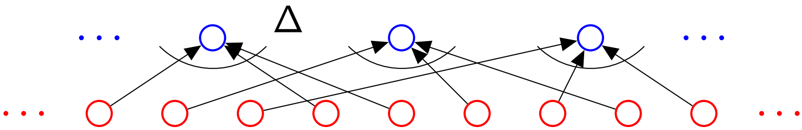

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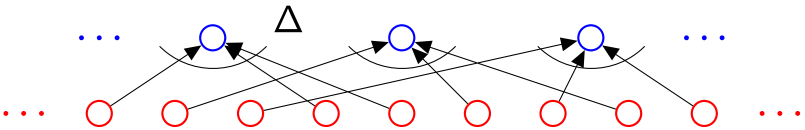

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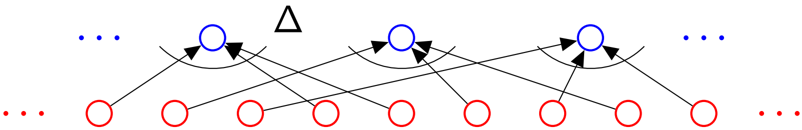

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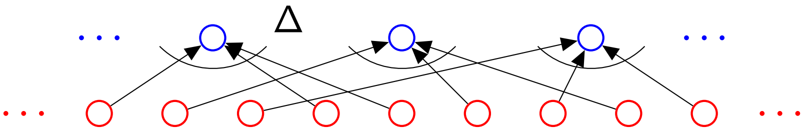

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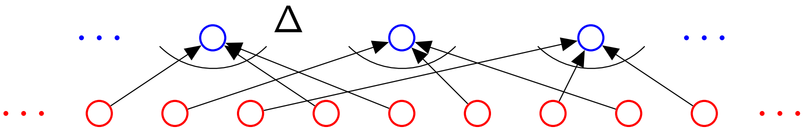

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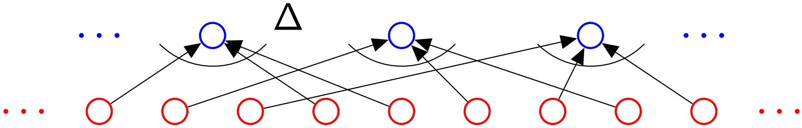

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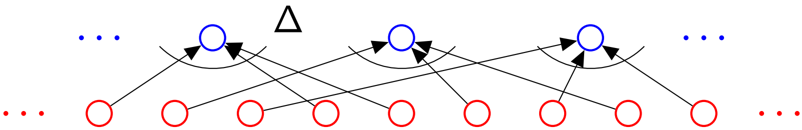

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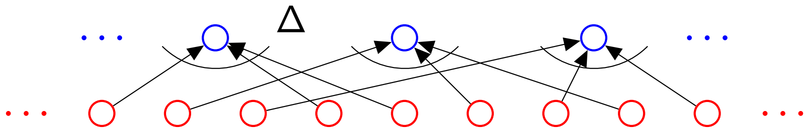

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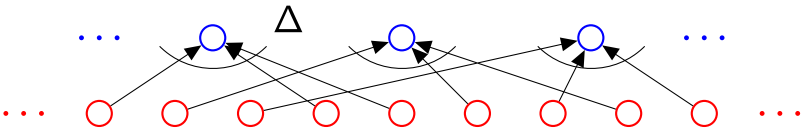

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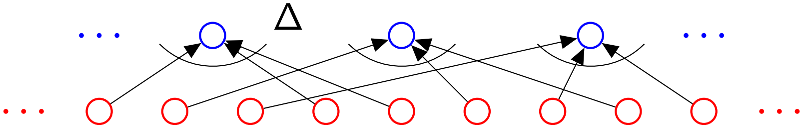

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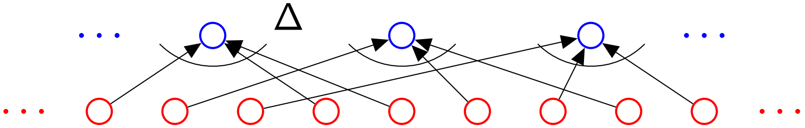

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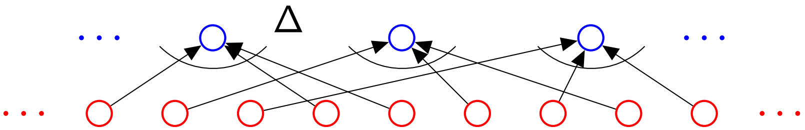

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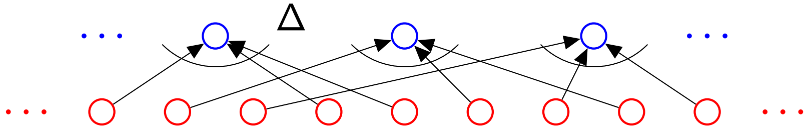

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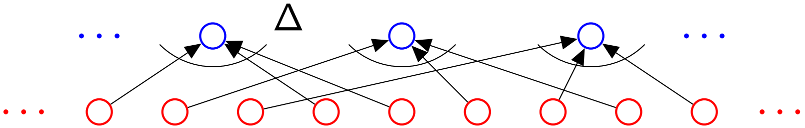

Orderings of ANAGRAM?

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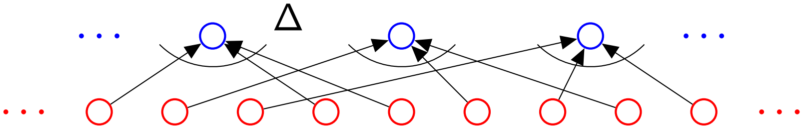

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Ordered Set: 7!

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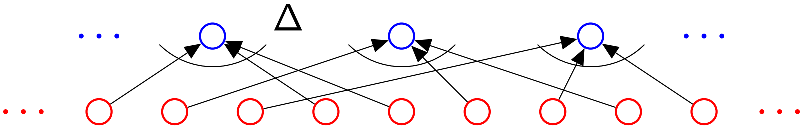

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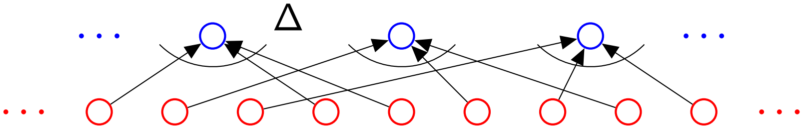

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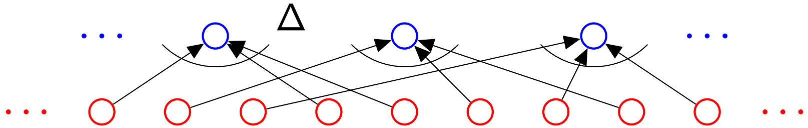

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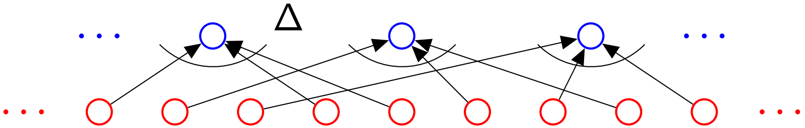

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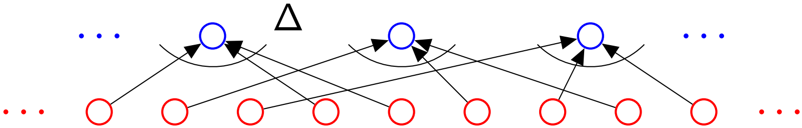

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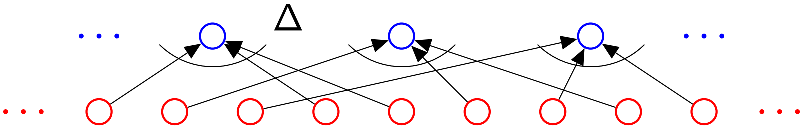

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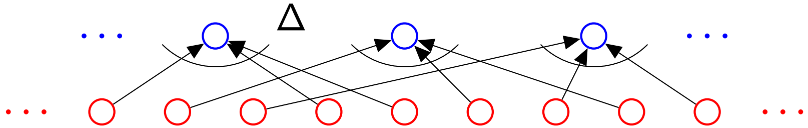

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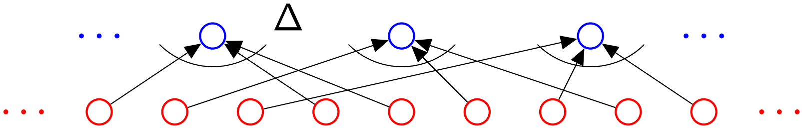

Orderings of ANAGRAM?
Ordered Set: 7! First rule.
A's are the same!
What is Δ?
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$A_1NA_2GRA_3M$, $A_2NA_1GRA_3M$, ...
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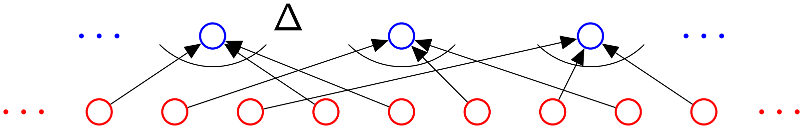

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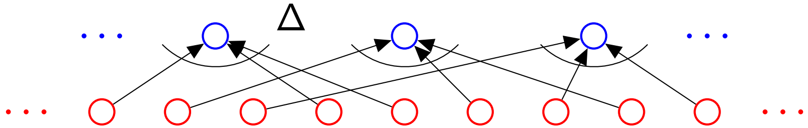

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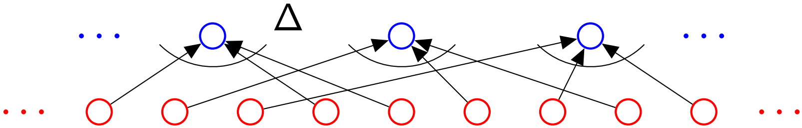

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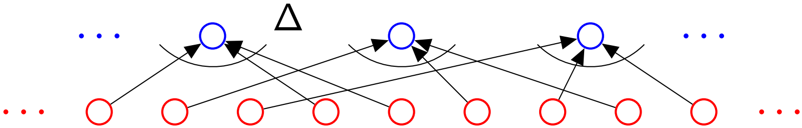

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Poll

Mark what’s correct.

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(A) $|\text{Poker hands}| = \binom{52}{5}$
(B) Orderings of ANAGRAM $= 7!/3!$
(C) Orderings of "CAT". $= 3!$
(D) Orders of MISSISSIPPI $= 11!/4!4!2!$
(E) Orderings of ANAGRAM $= 7!/4!$
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(A)-(E) are correct.

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How many orderings of letters of CAT?

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Ordered, except for A!

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total “extra counts” or orderings of three A’s?

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11 letters total.

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Sample k times from n objects with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.