Today.

Finish up counting.

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Thoughts on content...

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Thoughts on content...

...and midterm.

Some Practice.

How many orderings of letters of CAT?

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3 ways to choose first letter, 2 ways for second, 1 for last.

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$\implies 3 \times 2 \times 1$

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$\implies 3 \times 2 \times 1 = 3!$ orderings

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Ordered, except for A!

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total orderings of 7 letters.

Some Practice.

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total "extra counts" or orderings of three A's?

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4 S’s, 4 I’s, 2 P’s.
11 letters total.

Some Practice.

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Summary.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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Sample without replacement and order doesn't matter: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.
"n choose k"

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Sample k times from n objects with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

Sampling...

Sample k items out of n

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Without replacement:

Sampling...

Sample k items out of n

Without replacement:

Order matters:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1$

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Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

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Second Rule: divide by number of orders

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

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Second Rule: divide by number of orders – "$k!$"

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Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "k!"

$\implies \frac{n!}{(n-k)!k!}$.

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- Without replacement:
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"n choose k"

With Replacement.

Order matters: $n \times n$

Sampling...

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Without replacement:

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Order matters: $n \times n \times \ldots n$

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"n choose k"

With Replacement.

Order matters: $n \times n \times \ldots n = n^k$

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Problem: depends on how many of each item we chose!

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Order does not matter:

Second Rule: divide by number of orders – “$k!$”

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Different number of unordered elts map to each unordered elt.

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Unordered elt: $1,2,2$

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Unordered elt: $1, 2, 3$ — $3!$ ordered elts map to it.

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How do we deal with this

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Second Rule: divide by number of orders – "$k!$"

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Different number of unordered elts map to each unordered elt.

Unordered elt: $1,2,3$ $3!$ ordered elts map to it.
Unordered elt: $1,2,2$ $\frac{3!}{2!}$ ordered elts map to it.

How do we deal with this mess??

Splitting up some money....

How many ways can Bob and Alice split 5 dollars?

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For each of 5 dollars pick Bob or Alice(2^5), divide out order

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one ordered set: (B, B, B, B, B).

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"Sorted" way to specify, first Alice's dollars, then Bob's.

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(A, B, B, B, B):
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For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

4 for Bob and 1 for Alice:
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“Sorted” way to specify, first Alice’s dollars, then Bob’s.

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(A, B, B, B, B):
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and so on.

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one ordered set: (B, B, B, B, B).

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5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

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(B, B, B, B, B):
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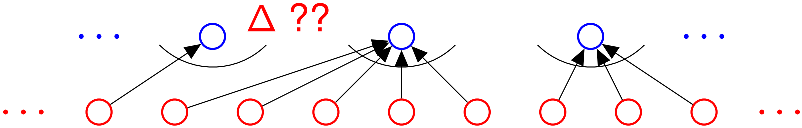
and so on.



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5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

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(B, B, B, B, B): 1: (B,B,B,B,B)
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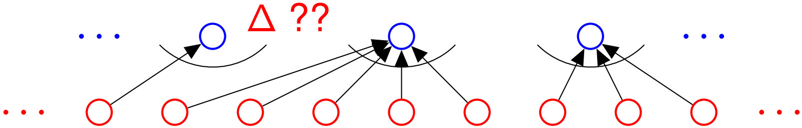

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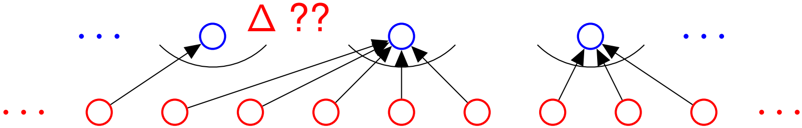
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

"Sorted" way to specify, first Alice's dollars, then Bob's.

(B, B, B, B, B): 1: (B,B,B,B,B)
(A, B, B, B, B): 5: (A,B,B,B,B),(B,A,B,B,B),(B,B,A,B,B),...
(A, A, B, B, B):

and so on.



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How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

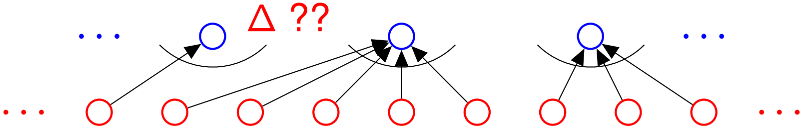
5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

4 for Bob and 1 for Alice:
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

"Sorted" way to specify, first Alice's dollars, then Bob's.

(B, B, B, B, B): 1: (B,B,B,B,B)
(A, B, B, B, B): 5: (A,B,B,B,B),(B,A,B,B,B),(B,B,A,B,B),...
(A, A, B, B, B): $\binom{5}{2}$;(A,A,B,B,B),(A,B,A,B,B),(A,B,B,A,B),...

and so on.



Second rule of counting is no good here!

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Splitting 5 dollars..

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Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

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Separate Alice's dollars from Bob's and then Bob's from Eve's.

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Stars and Bars: $\star\star|\star|\star\star$.

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Each sequence of stars and bars "is" a split.

Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

Stars and Bars.

How many different 5 star and 2 bar diagrams?

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Bars in second and seventh position.

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Bars in first and third position.

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$\binom{7}{2}$ ways to do so and
$\binom{7}{2}$ ways to split 5 dollars among 3 people.

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Ways to add up n numbers to sum to k?

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Or: k unordered choices from set of n possibilities with replacement.
Sample with replacement where order doesn't matter.

Counting basics.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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One-to-one rule: equal in number if one-to-one correspondence.

Sample with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

Bijection: sums to 'k' $\rightarrow$ stars and bars.

$$S = \{(n_1, n_2, n_3) : n_1 + n_2 + n_3 = 5\}$$

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$|S| = |T| = \binom{7}{2}$.

Stars and Bars Poll

Mark whats correct.

(A) ways to split n dollars among k: $\binom{n+k-1}{k-1}$

(B) ways to split k dollars among n: $\binom{k+n-1}{n-1}$

(C) ways to split 5 dollars among 3: $\binom{7}{5}$

(D) ways to split 5 dollars among 3: $\binom{5+3-1}{3-1}$

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All correct.

Balls in bins.

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Example: 5 digit numbers.

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Example: Poker hands.

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Example: Poker hands.

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5 balls into 10 bins
5 samples from 10 possibilities with replacement
Example: 5 digit numbers.

5 indistinguishable balls into 52 bins only one ball in each bin
5 samples from 52 possibilities without replacement
Example: Poker hands.

5 indistinguishable balls into 3 bins
5 samples from 3 possibilities with replacement and no order

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5 balls into 10 bins
5 samples from 10 possibilities with replacement
Example: 5 digit numbers.

5 indistinguishable balls into 52 bins only one ball in each bin
5 samples from 52 possibilities without replacement
Example: Poker hands.

5 indistinguishable balls into 3 bins
5 samples from 3 possibilities with replacement and no order
Dividing 5 dollars among Alice, Bob and Eve.

Poll

Mark whats correct.

k Balls in n bins.

dis == distinguishiable
unique = one ball in each bin.

(A) dis $=>$ n^k
(B) dis,unique $=>$ $n!/(n-k)!$
(C) indis, unique $=>$ $\binom{n}{k}$
(D) dis, $=>$ $n!/(n-k)!$
(E) indis, $=>$ $\binom{n+k-1}{k-1}$
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Two indistinguishable jokers in 54 card deck.
How many 5 card poker hands?

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How many 5 card poker hands? Choose 4 cards plus one of 2 jokers!

$$\binom{52}{5} + 2 * \binom{52}{4} +$$

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Theorem: $\binom{54}{5}$

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Theorem: $\binom{54}{5} = \binom{52}{5} + 2 * \binom{52}{4} + \binom{52}{3}$.

Algebraic Proof:

Sum Rule

Two indistinguishable jokers in 54 card deck.
How many 5 card poker hands?
Sum rule: Can sum over disjoint sets.
No jokers "exclusive" or One Joker "exclusive" or Two Jokers

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and what's left out

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Theorem: ${n \choose k} = {n \choose n-k}$

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How many subsets of size k?
Choose a subset of size $n-k$
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Choosing a subset of size k is same

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Pascal's Triangle

Pascal's Triangle

	0	
1		1

Pascal’s Triangle

		0		
	1		1	
1		2		1

Pascal's Triangle

			0			
		1		1		
	1		2		1	
1		3		3		1

Pascal's Triangle

				0				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

Pascal's Triangle

				0				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

Pascal's Triangle

				0				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

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Foil (4 terms)

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Foil (4 terms) on steroids:

Pascal's Triangle

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2^n terms: choose 1 or x from each term $(1+b)$.

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0
1 1
1 2 1
1 3 3 1
1 4 6 4 1

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Simplify: collect all terms corresponding to x^k.

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1 1
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Coefficient of x^k $\binom{n}{k}$: choose k terms with x in product.

Pascal's Triangle

$$
\begin{array}{ccccccccc}
 & & & & 0 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
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$$
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$$

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0

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Pascal's rule $\Longrightarrow$ $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

Combinatorial Proofs.

Theorem: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

Proof: How many size k subsets of $n+1$?

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How many contain the first element?

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How many contain the first element?
Chose first element, need $k-1$ more from remaining n elements.

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How many don't contain the first element ?

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Need to choose k elements from remaining n elts.

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Sum Rule: size of union of disjoint sets of objects.

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Without and with first element $\rightarrow$ disjoint.

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So, $\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$. $\square$

Combinatorial Proof.

Theorem: $\binom{n}{k} = \binom{n-1}{k-1} + \cdots + \binom{k-1}{k-1}$.

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$$\{1, \ldots, {\color{blue}i}, \underline{\ldots, n}\}$$

Must choose $k-1$ elements from $n-i$ remaining elements.

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Add them up to get the total number of subsets of size k
which is also $\binom{n+1}{k}$.

□

Binomial Theorem: $x = 1$

Theorem: $2^n = \binom{n}{n} + \binom{n}{n-1} + \cdots + \binom{n}{0}$

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Construct a subset with sequence of n choices:

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Sum over i to get total number of subsets..

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Sum over i to get total number of subsets..which is also 2^n. □

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S and T, $|S \cup T| = |S| + |T|$
Used to reason about all subsets
by adding number of subsets of size 1, 2, 3,. . .

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Used to reason about all subsets
by adding number of subsets of size 1, 2, 3,...

Also reasoned about subsets that contained
or didn't contain an element. (E.g., first element, first i elements.)

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S and T, $|S \cup T| = |S| + |T|$
Used to reason about all subsets
by adding number of subsets of size 1, 2, 3,...

Also reasoned about subsets that contained
or didn't contain an element. (E.g., first element, first i elements.)

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For any S and T, $|S \cup T| = |S| + |T| - |S \cap T|$.

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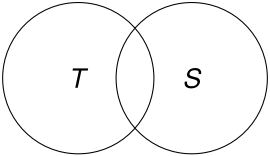

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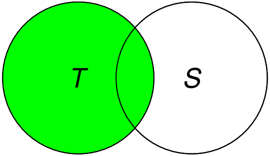

In T. $\implies |T|$

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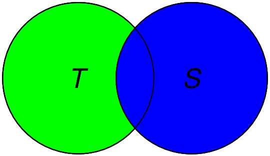

In T. $\implies |T|$
In S. $\implies + \; |S|$

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S **and** T**,** $|S \cup T| = |S| + |T|$
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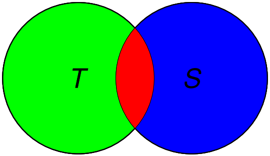

$S \cap T$

In T. $\implies$ $|T|$
In S. $\implies$ $+ \; |S|$
Elements in $S \cap T$ are counted twice.

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S and T, $|S \cup T| = |S| + |T|$
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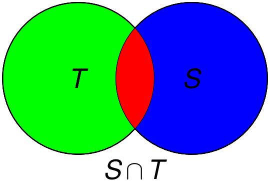

In T. $\implies |T|$
In S. $\implies + \ |S|$
Elements in $S \cap T$ are counted twice.
Subtract. $\implies -|S \cap T|$

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S and T, $|S \cup T| = |S| + |T|$
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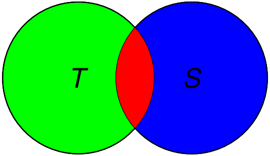

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Elements in $S \cap T$ are counted twice.
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$$|S \cup T| = |S| + |T| - |S \cap T|$$

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Example: How many 10-digit phone numbers have 7 as their first or second digit?

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S = phone numbers with 7 as first digit.

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Example: How many 10-digit phone numbers have 7 as their first or second digit?

S = phone numbers with 7 as first digit. $|S| = 10^9$

Simple Inclusion/Exclusion

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T = phone numbers with 7 as second digit.

Simple Inclusion/Exclusion

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T = phone numbers with 7 as second digit. $|T| = 10^9$.

$S \cap T$ = phone numbers with 7 as first and second digit. $|S \cap T| = 10^8$.

Answer: $|S| + |T| - |S \cap T| = 10^9 + 10^9 - 10^8$.

Inclusion/Exclusion

$|A_1 \cup \cdots \cup A_n| =$
$\sum_i |A_i| - \sum_{i,j} |A_i \cap A_j| + \sum_{i,j,k} |A_i \cap A_j \cap A_k| \cdots (-1)^n |A_1 \cap \cdots A_n|$.

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Total: 3 - 3 + 1 = 1.

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Formulaically: x is in intersection of three sets.

Inclusion/Exclusion

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x subtracted once in terms: $|A_1 \cap A_3|$, $|A_1 \cap A_2|$, $|A_2 \cap A_3|$.
x added once in $|A_1 \cap A_2 \cap A_3|$.
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$|A_1 \cup \cdots \cup A_n| =$
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Total: 2 -1 = 1.

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$\binom{3}{3}$ for terms of form $|A_i \cap A_j \cap A_k|$.

Total: $\binom{3}{1} - \binom{3}{2} + \binom{3}{3} = 1$.

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$|A_1 \cup \cdots \cup A_n| =$
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Idea: how many times is each element counted?
  Element x in m sets: $x \in A_{i_1} \cap A_{i_2} \cdots \cap A_{i_m}$.

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Counted $\binom{m}{i}$ times in ith summation.

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Binomial Theorem:
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Inclusion/Exclusion

$|A_1 \cup \cdots \cup A_n| =$
$\sum_i |A_i| - \sum_{i,j} |A_i \cap A_j| + \sum_{i,j,k} |A_i \cap A_j \cap A_k| \cdots (-1)^n |A_1 \cap \cdots A_n|$.

Idea: how many times is each element counted?

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Each element counted once!

Summary.

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Count with order:

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Disjoint – so add!

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth?

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What is the minimum I need to know (remember) to know stuff.

Radius of the earth? Distance to the sun?

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Conceptualization.
Reason and understand an argument and you can generate a lot.

Calculus

What is the first half of calculus about?

Calculus

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The slope of a tangent line to a function at a point.

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Plugin definitions (e.g., $\lim_{h \to 0} \cdots$), make argument (or do derivation.)

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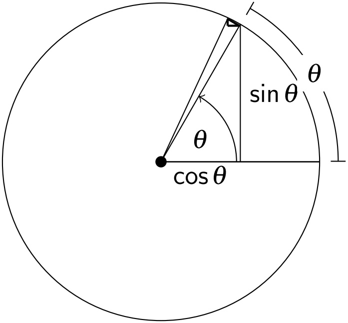

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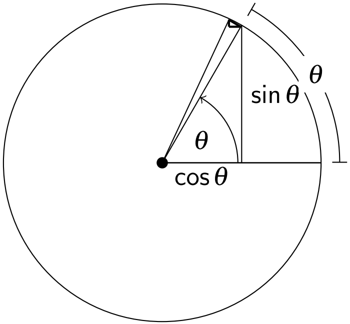

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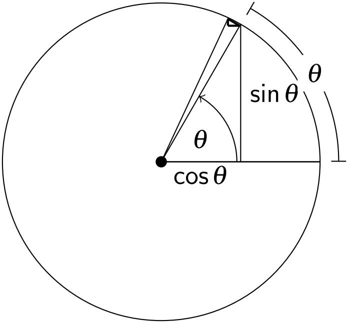

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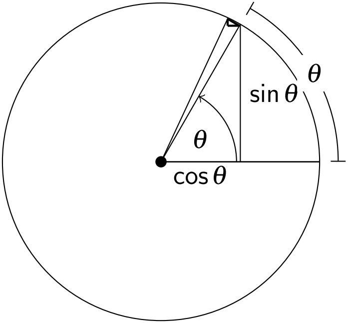

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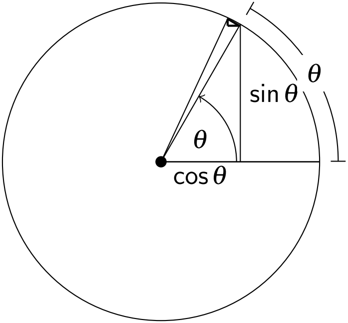

Rise. Similar triangle!!!
Rise of sine $\propto$ cosine!
Change of cosine $\propto$ -sine.

Fundamental Theorem of Calculus.

Conceptual: Height times Width = Area.

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Useful?

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Application Math: Newton.
Velocity times Time is Distance.

$\int_a^b f(x)d(x)$ “is” area under the curve.

Arguments, reasoning.

What you know: slope, limit.

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Knowing how to program plus some syntax (google) gives the ability to program.

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...plus reasoning.

CS 70 : ideas.

Induction

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Induction $\equiv$ every integer has a next one.

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Induction $\equiv$ every integer has a next one. Graph theory.

Number of edges is sum of degrees.

$\Delta+1$ coloring. Neighbors only take up Δ.

Connectivity plus connected components.

Eulerian paths: if you enter you can leave.

Euler's formula: tree has $v-1$ edges and 1 face plus
each extra edge makes additional face.

$v-1+(f-1)=e$

CS 70 : ideas.

Number theory.

A divisor of x and y divides $x - y$.

The remainder is always smaller than the divisor.

$\implies$ Euclid's GCD algorithm.

Multiplicative Inverse.

Fermat's theorem from function with inverse is a bijection.

Gives RSA.

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Error Correction.
(Any) Two points determine a line.
(well, and d points determine a degree $d+1$-polynomials.
Cuz, factoring.
Find line by linear equations.
If a couple are wrong, then multiply them by zero, i.e., Error polynomial.