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Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x)$

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Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False .

Back to: Wason's experiment:1

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$Flew(B)$ = False .

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So $Chicago(Bob)$ must be False .

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$Chicago(C)$ = True .

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Yes.

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Only have to turn over cards for Bob and Charlie.

Review.

Theory: If you drink alcohol you must be at least 18.

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Theory: If you drink alcohol you must be at least 18.

Which cards do you turn over?

Drink Alcohol $\implies$ "≥ 18"

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Propositional Forms:

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Propositional Forms: $\wedge$,$\vee$, $\neg$, $P \implies Q \equiv \neg P \vee Q$.

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Truth Table. Putting together identities. (E.g., cases, substitution.)

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Predicates, $P(x)$, and quantifiers. $\forall x, P(x)$.

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DeMorgan's: $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$.

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Predicates, $P(x)$, and quantifiers. $\forall x, P(x)$.

DeMorgan's: $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$. $\neg \forall x, P(x) \equiv \exists x, \neg P(x)$.

CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

If time: discuss induction.

Last time: Existential statement.

How to prove existential statement?

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Give an example. (Sometimes called ”proof by example.”)

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Often used to disprove claim.

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Give an example. (Sometimes called ”proof by example.”)

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Often used to disprove claim.

Homework.

Quick Background and Notation.

Integers closed under addition.

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$a, b \in Z \implies a + b \in Z$

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$7|23$?

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$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

$4|2$? No!

$2|-4$? Yes!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

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3|15

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$3|15$ since for $q = 5$,

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Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$, $15 = 3(5)$.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a+b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes! Since for $q = 2$, $4 = (2)2$.

$7|23$? No!

$4|2$? No!

$2|-4$? Yes! Since for $q = 2$, $-4 = (-2)2$.

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Divides.

$a|b$ means

(A) There exists $k \in \mathbb{Z}$, with $a = kb$.

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Another direct proof.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer. $\implies 11|n$. $\square$

Direct proof of $P \implies Q$:

Another direct proof.

Let D_3 be the 3 digit natural numbers.

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Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

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Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

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Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. □

Proof of lemma: Assume a solution of the form a/b.

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Multiply by b^5,

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The fourth case is the only one possible, so the lemma follows. □

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Question: Which case holds? Don't know!!!

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(B) I don't know.

Be careful.

Theorem: $3 = 4$

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Don't assume what you want to prove!

Be really careful!

Theorem: $1 = 2$
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\begin{aligned}
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$P \implies Q$ does not mean $Q \implies P$.

Summary: Note 2.

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Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.
or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Careful when proving!

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

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Don't assume the theorem. Divide by zero.

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Don't assume the theorem. Divide by zero.Watch converse. ...

CS70: Note 3. Induction!

Poll. What's the biggest number?

(A) 100

(B) 101

(C) n+1

(D) infinity.

(E) This is about the "recursive leap of faith."