Left over

Finishing stable matching...

Good for jobs? candidates?

Is the Job-Proposes better for jobs?

Good for jobs? candidates?

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Good for jobs? candidates?

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Is it possible:

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b-optimal pairing different from the b'-optimal matching!

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Yes?

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Understanding Optimality: by example.

A:	1,2	1:	A,B
B:	1,2	2:	B,A

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A:	1,2	1:	A,B
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Consider pairing: $(A,1),(B,2)$.

Understanding Optimality: by example.

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Consider pairing: $(A, 1), (B, 2)$.

Stable?

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Stable? Yes.

Understanding Optimality: by example.

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Stable? Yes.

Optimal for B?

Understanding Optimality: by example.

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Pairing S: $(A,1),(B,2)$. Stable? Yes.

Pairing T: $(A,2),(B,1)$.

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Which is optimal for A?

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Which is optimal for A? S Which is optimal for B? S
Which is optimal for 1?

Understanding Optimality: by example.

A:	1,2	1:	A,B
B:	1,2	2:	B,A

Consider pairing: $(A,1),(B,2)$.

Stable? Yes.

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Which is optimal for 1? T

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Pairing *T*: $(A,2),(B,1)$. Also Stable.

Which is optimal for *A*? *S* Which is optimal for *B*? *S*
Which is optimal for 1? *T* Which is optimal for 2?

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Which is optimal for A? S Which is optimal for B? S
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Pessimality?

Job Propose and Candidate Reject is optimal?

For jobs?

Job Propose and Candidate Reject is optimal?

For jobs? For candidates?

Job Propose and Candidate Reject is optimal?

For jobs? For candidates?

Theorem: Job Propose and Reject produces a job-optimal pairing.

Job Propose and Candidate Reject is optimal?

For jobs? For candidates?

Theorem: Job Propose and Reject produces a job-optimal pairing.

Proof:

Job Propose and Candidate Reject is optimal?

For jobs? For candidates?

Theorem: Job Propose and Reject produces a job-optimal pairing.

Proof:
Assume not:

Job Propose and Candidate Reject is optimal?

For jobs? For candidates?

Theorem: Job Propose and Reject produces a job-optimal pairing.

Proof:
Assume not: there is a job b does not get optimal candidate, g.

Job Propose and Candidate Reject is optimal?

For jobs? For candidates?

Theorem: Job Propose and Reject produces a job-optimal pairing.

Proof:
Assume not: there is a job b does not get optimal candidate, g.

There is a stable pairing S where b and g are paired.

Job Propose and Candidate Reject is optimal?

For jobs? For candidates?

Theorem: Job Propose and Reject produces a job-optimal pairing.

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Let t be first day job b gets rejected

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Let t be first day job b gets rejected
by its optimal candidate g who it is paired with

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Job Propose and Candidate Reject is optimal?

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b^* - knocks b off of g's string on day t

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b^* - knocks b off of g's string on day $t \implies g$ prefers b^* to b

Job Propose and Candidate Reject is optimal?

For jobs? For candidates?

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Proof:
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By choice of t, b^* likes g at least as much as optimal candidate.

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$\implies b^*$ prefers g to its partner g^* in S.

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By choice of t, b^* likes g at least as much as optimal candidate.

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Rogue couple for S.

Job Propose and Candidate Reject is optimal?

For jobs? For candidates?

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By choice of t, b^* likes g at least as much as optimal candidate.

$\implies b^*$ prefers g to its partner g^* in S.

Rogue couple for S.
So S is not a stable pairing.

Job Propose and Candidate Reject is optimal?

For jobs? For candidates?

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Structural statement: Job optimality $\implies$ Candidate pessimality.

Quick Questions.

How does one make it better for candidates?

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Propose and Reject - stable matching algorithm. One side proposes.

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Candidates propose.

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Candidates propose. $\implies$ optimal for candidates.

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The method was used to match residents to hospitals.

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Improvement Lemma plus every day the job gets to choose.
Optimality proof:
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that is, no rogue couple by improvement, job choice, and well ordering principle. Candidate Pessimality:
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Lecture 5: Graphs.

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Definitions: model.

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Fact!

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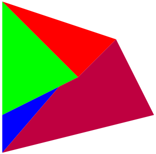
Map Coloring.



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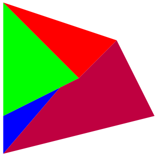
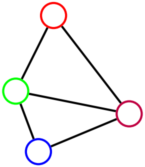
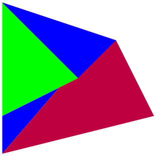
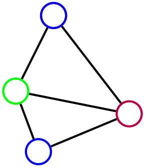




Fewer Colors?

Map Coloring.





Yes! Three colors.

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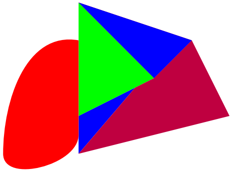

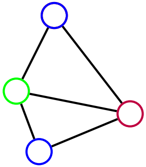

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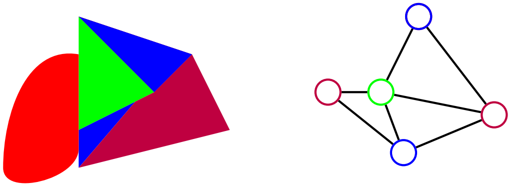

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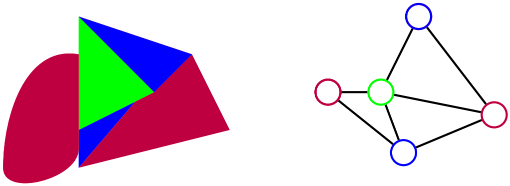

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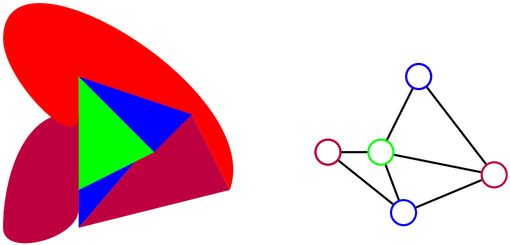

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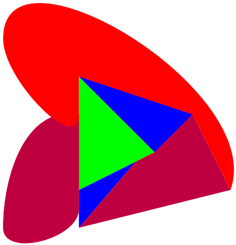

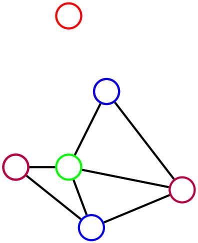

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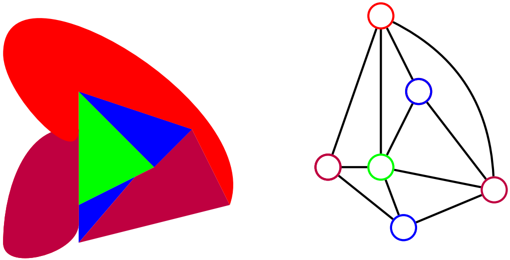

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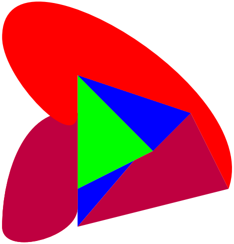

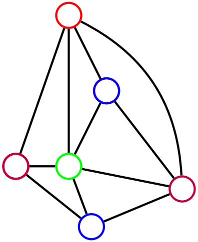

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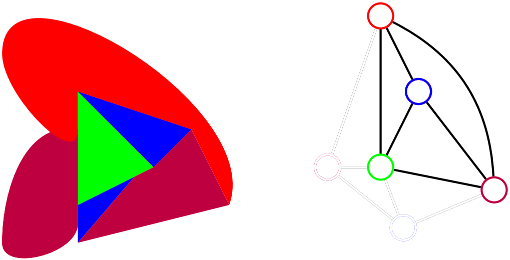

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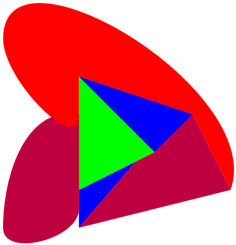
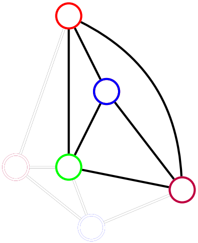
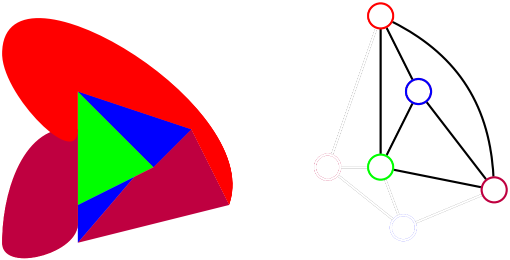




Four colors required!

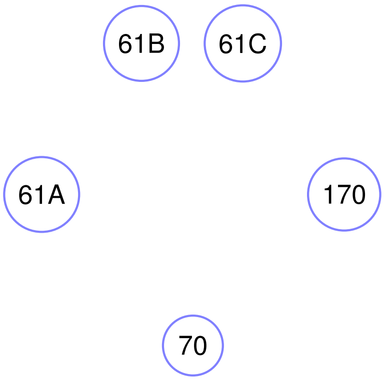
Map Coloring.



Four colors required!

Theorem: Four colors enough for maps on the plane.

Scheduling: coloring.



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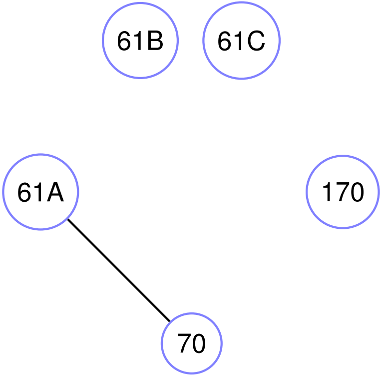

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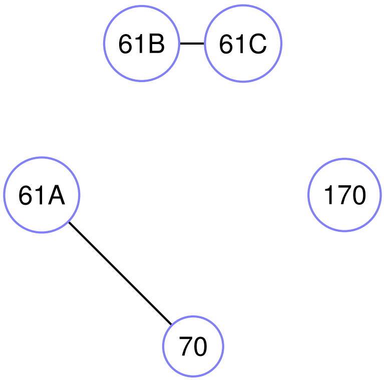

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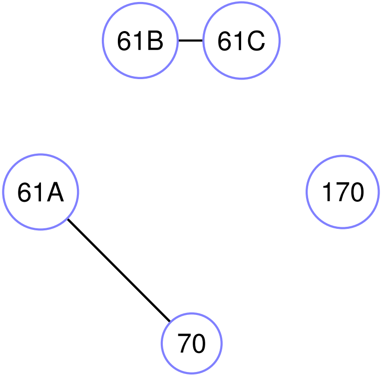

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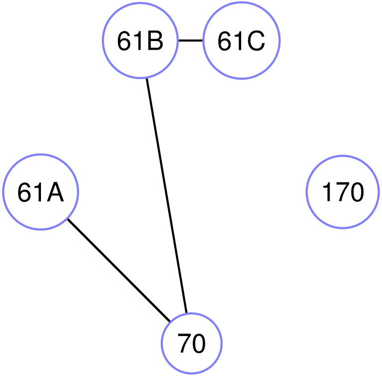

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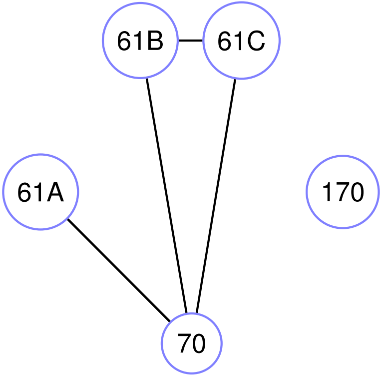

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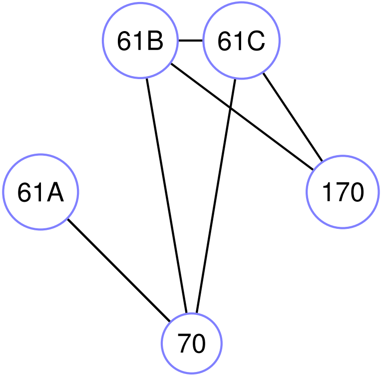

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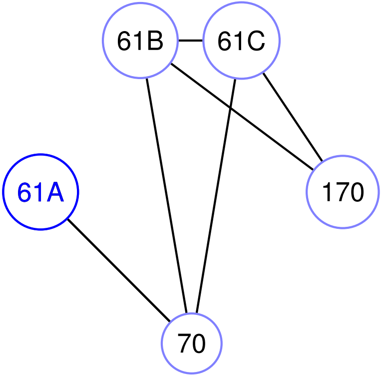

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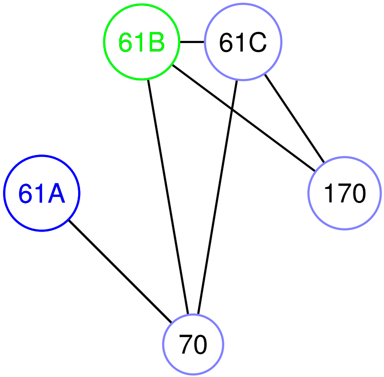

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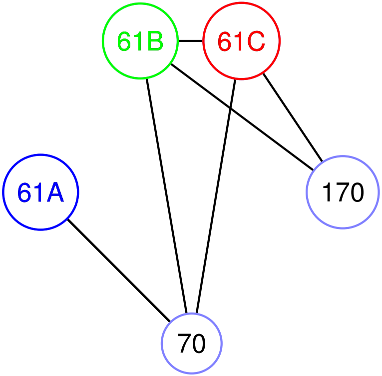

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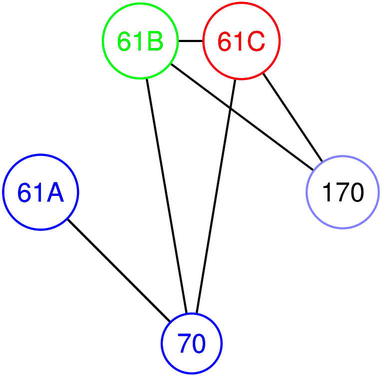
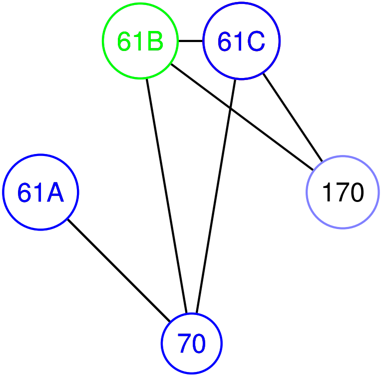
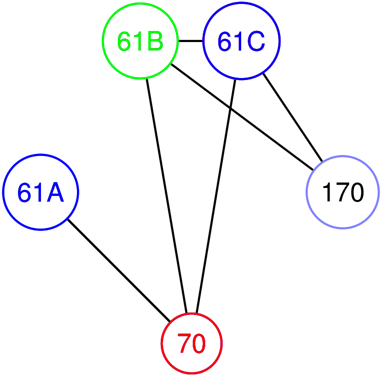


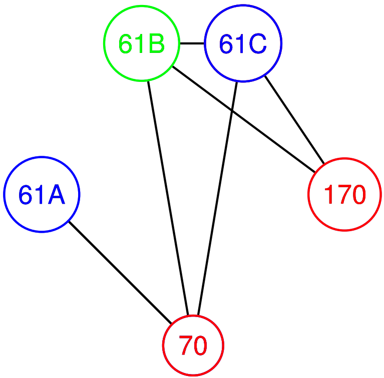
Scheduling: coloring.



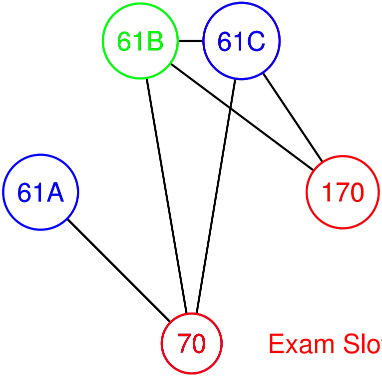
Scheduling: coloring.



Scheduling: coloring.



Scheduling: coloring.

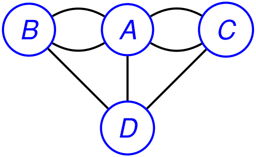


Exam Slot 1.

Exam Slot 2.

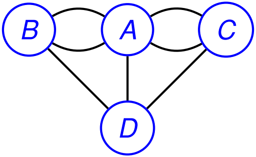
Exam Slot 3.

Graphs: formally.



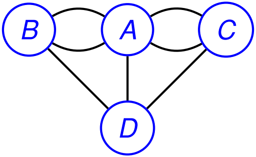
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

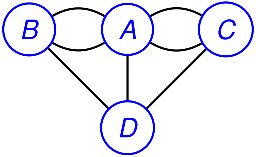
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.

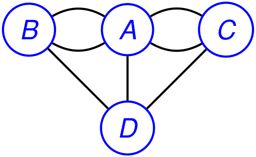


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

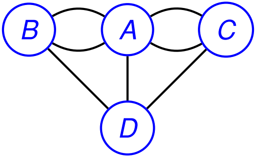
Graphs: formally.



Graph: $G = (V, E)$.

- V - set of vertices.
  - $\{A, B, C, D\}$
- $E \subseteq V \times V$ -

Graphs: formally.



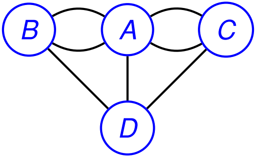
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

Graphs: formally.



Graph: $G = (V, E)$.

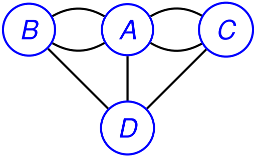
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

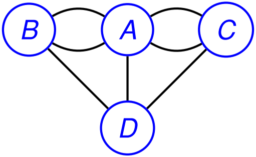
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

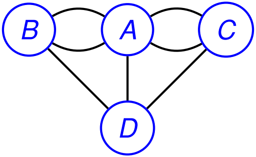
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\},$

Graphs: formally.



Graph: $G = (V, E)$.

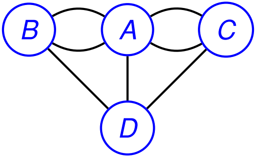
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A,B\},\{A,B\},\{A,C\},\{A,C\},\{B,D\},\{A,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.

Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

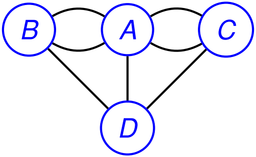
$E \subseteq V \times V$ - set of edges.

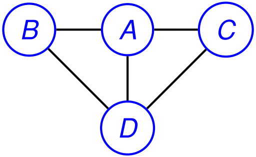
$\{\{A,B\},\{A,B\},\{A,C\},\{A,C\},\{B,D\},\{A,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

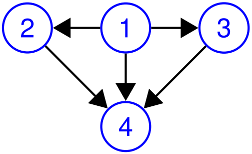
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

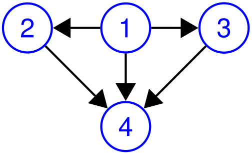
Multigraph above.

Directed Graphs



$G = (V, E)$.

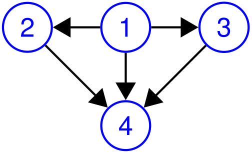
Directed Graphs



$G = (V, E)$.

V - set of vertices.

Directed Graphs

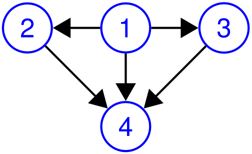


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



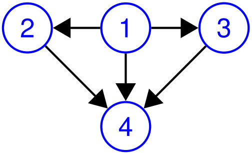
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



$G = (V, E)$.

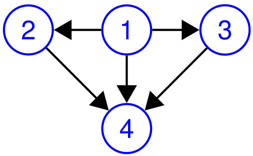
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



$G = (V, E)$.

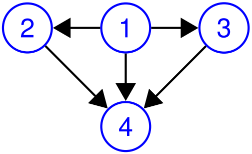
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),$

Directed Graphs



$G = (V, E)$.

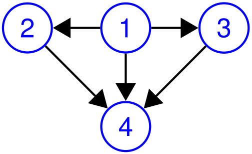
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3), (1,4),$

Directed Graphs



$G = (V, E)$.

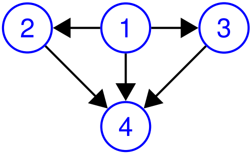
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

Directed Graphs



$G = (V, E)$.

V - set of vertices.

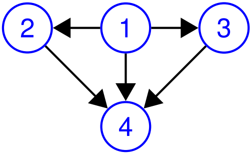
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.

Directed Graphs



$G = (V, E)$.

V - set of vertices.

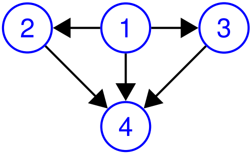
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament:

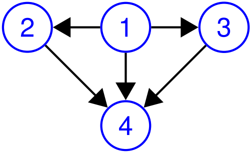
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1,2,3,4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2,

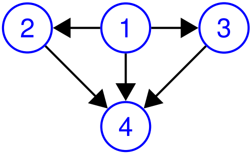
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence:

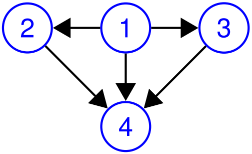
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



$G = (V, E)$.

V - set of vertices.

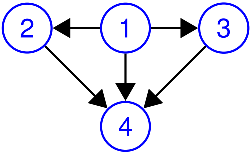
$\{1,2,3,4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Directed Graphs



$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

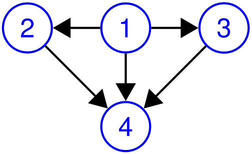
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:

Directed Graphs

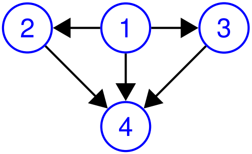


$G = (V, E)$.
V - set of vertices.
$\{1,2,3,4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed?

Directed Graphs



$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

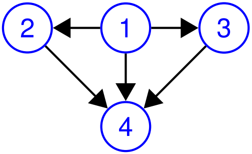
E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?

Directed Graphs

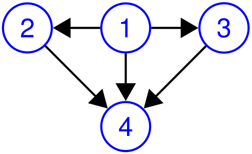


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends.

Directed Graphs

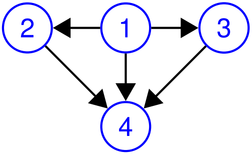


$G = (V, E)$.
V - set of vertices.
$\{1,2,3,4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.

Directed Graphs



$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

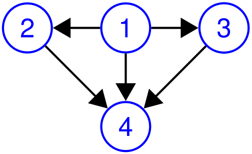
E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes.

Directed Graphs

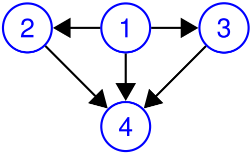


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

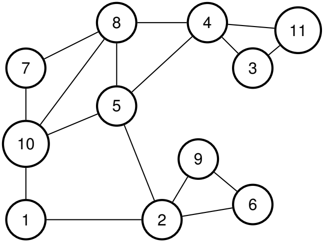
Graph: $G = (V, E)$

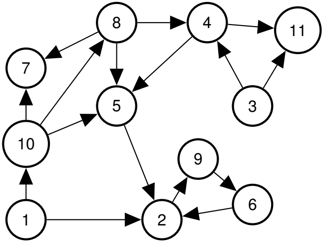
neighbors, adjacent, degree, incident, in-degree, out-degree

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



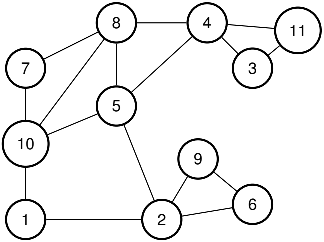


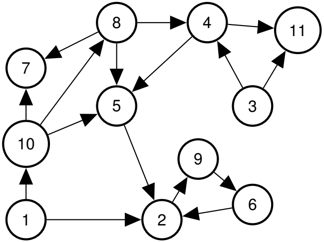
Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



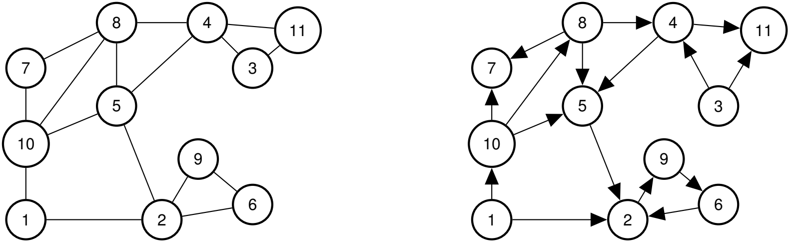


Neighbors of 10? 1,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

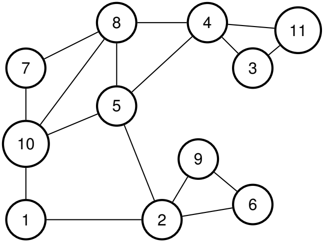


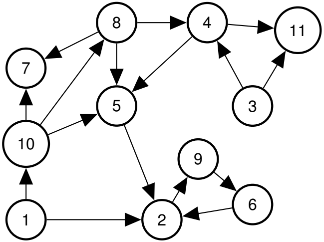
Neighbors of 10? 1,5,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



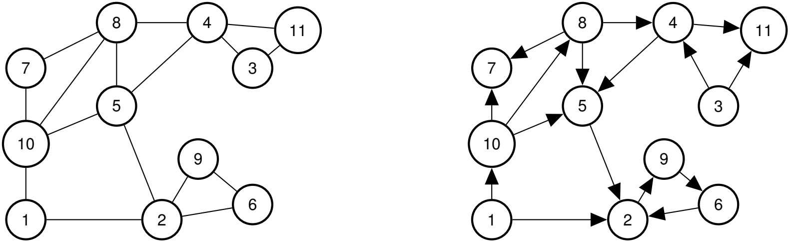


Neighbors of 10? 1,5,7,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

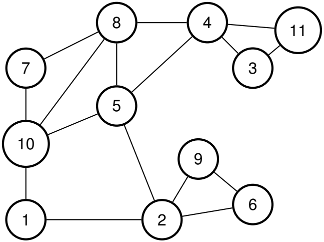


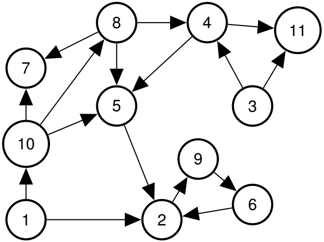
Neighbors of 10? 1,5,7, 8.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





Neighbors of 10? 1,5,7, 8.

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Graph Concepts and Definitions.

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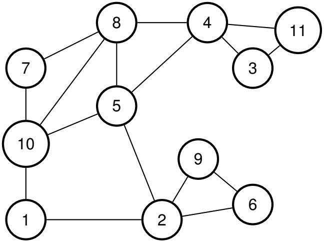
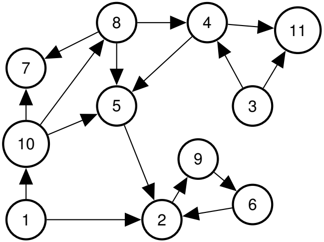
neighbors, adjacent, degree, incident, in-degree, out-degree





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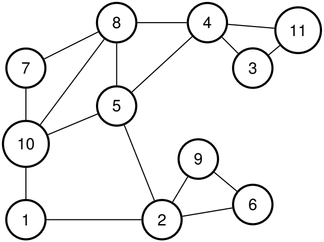

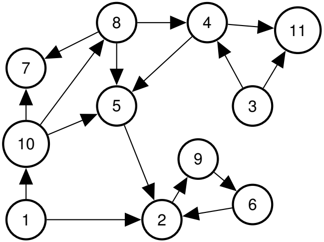

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Degree of vertex 1?

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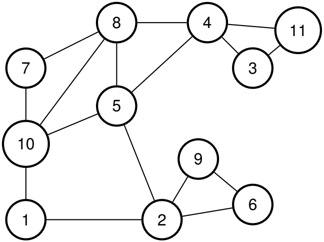

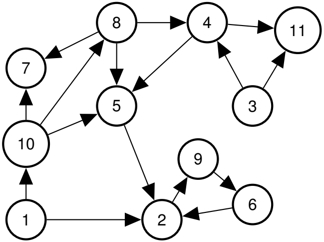

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Degree of vertex 1? 2

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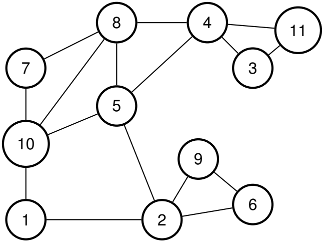

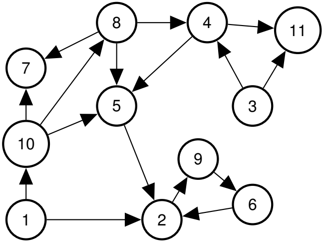

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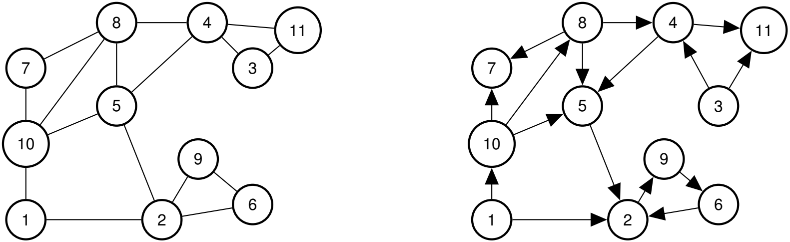

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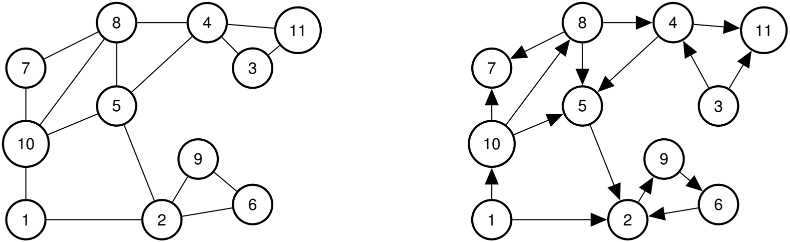

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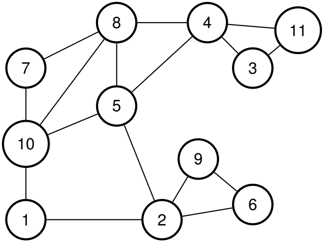

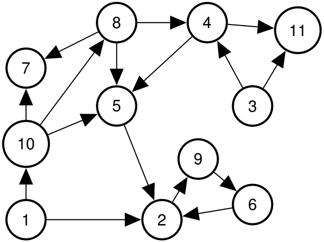

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Directed graph?

Graph Concepts and Definitions.

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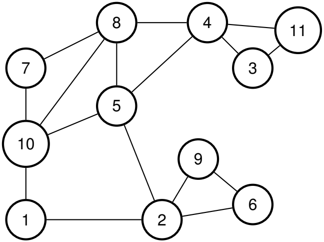

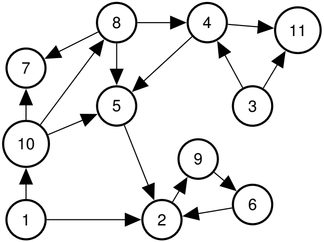

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Directed graph?

In-degree of 10?

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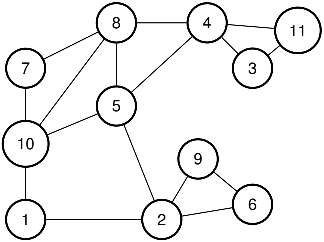

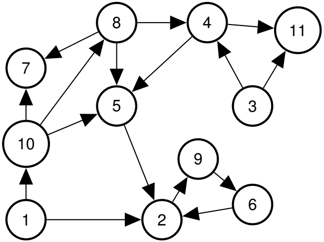

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Directed graph?

In-degree of 10? 1

Graph Concepts and Definitions.

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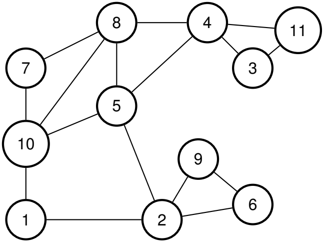

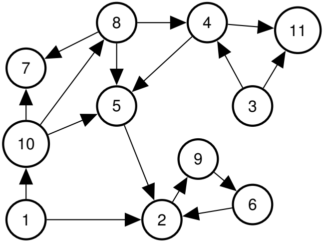

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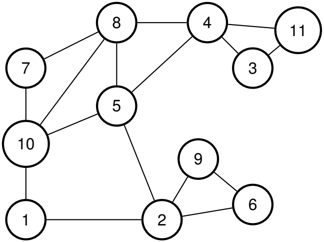

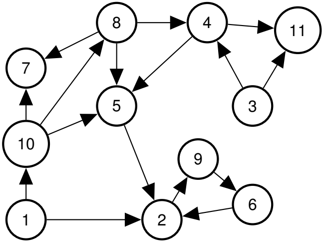

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Equals number of neighbors in simple graph.

Directed graph?

In-degree of 10? 1   Out-degree of 10? 3

Graph Concepts and Definitions.

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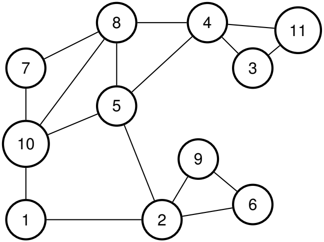

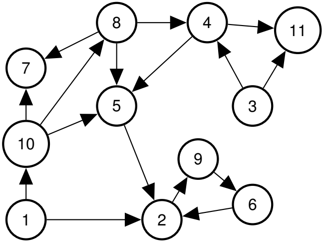

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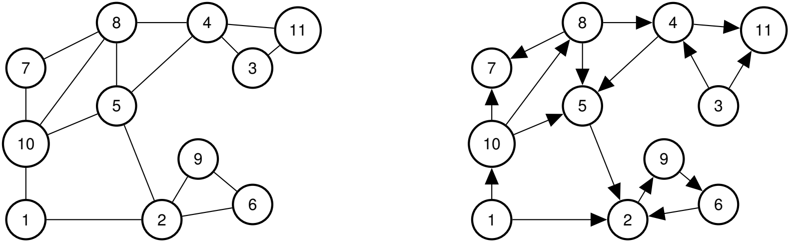

Edge $(8, 5)$ **is incident to:**

(A) Vertex 8.
(B) Vertex 5.
(C) Edge $(8, 5)$.
(D) Edge $(8, 4)$.
(E) Vertex 10.

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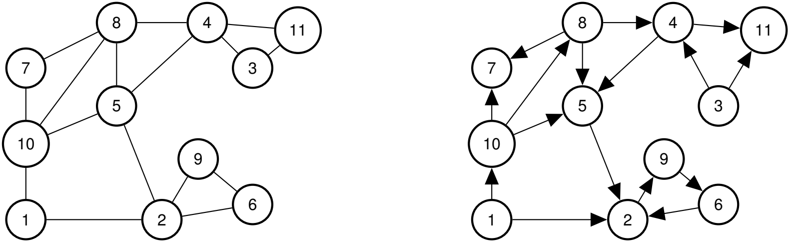

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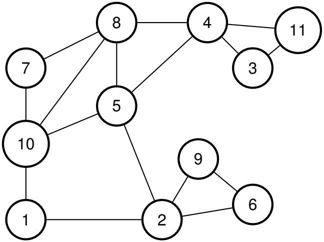

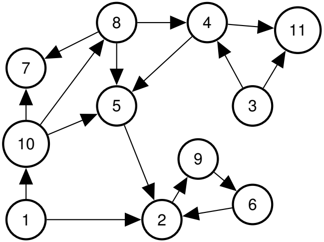

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The degree of a vertex is:

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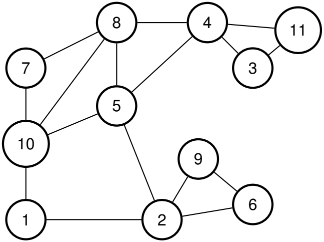

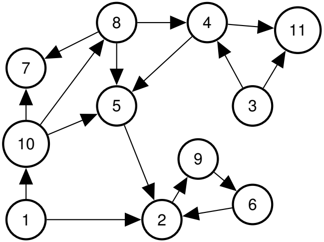

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The sum of the vertex degrees is equal to

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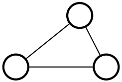
Not (A)!



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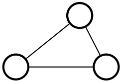
Not (A)! Triangle.



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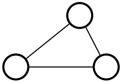

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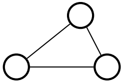

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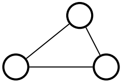

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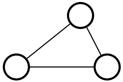

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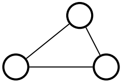

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What? For triangle number of edges is 3, the sum of degrees is 6.

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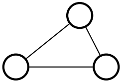

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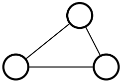

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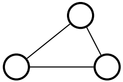

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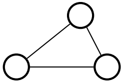

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(B) $2|V|$?
(A) is true.

Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Quick Proof of an Equality.

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Recall:

Quick Proof of an Equality.

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edge, (u, v), is incident to endpoints, u and v.

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Let's count incidences in two ways.

Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

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Total Incidences? The sum over vertices of degrees!

Thm: Sum of vertex degress is $2|E|$.

Poll: Proof of “handshake” lemma.

What’s true?

(A) The number of edge-vertex incidences for an edge e is 2.
(B) The total number of edge-vertex incidences is $|V|$.
(C) The total number of edge-vertex incidences is $2|E|$.
(D) The number of edge-vertex incidences for a vertex v is its degree.
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.

Poll: Proof of “handshake” lemma.

What’s true?

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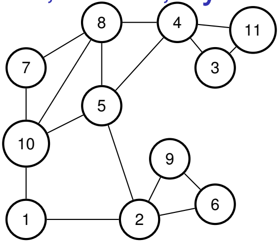
(F) The total number of edge-vertex incidences is the sum of the degrees.

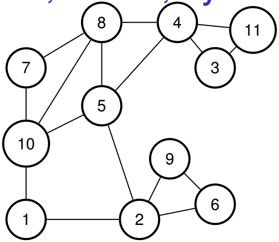
(A),(C), (D), (E), and (F).

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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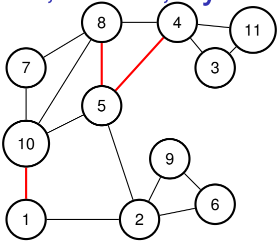

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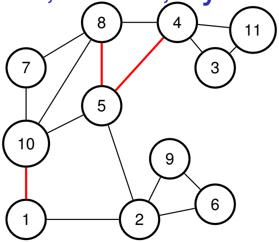
Path? $\{1, 10\}$, $\{8, 5\}$, $\{4, 5\}$?

Paths, walks, cycles, tour.



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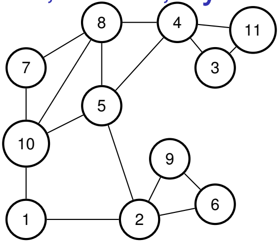

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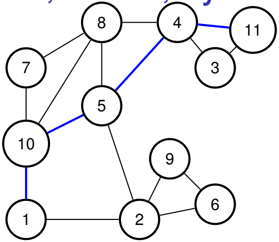

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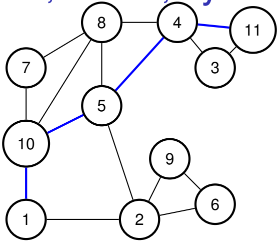

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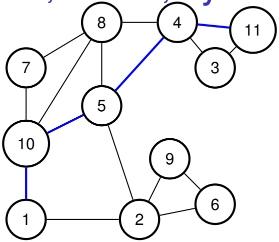

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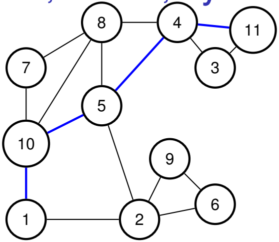

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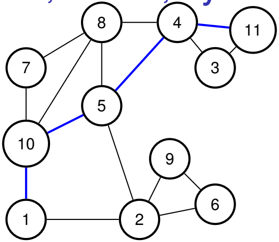
Quick Check!

Paths, walks, cycles, tour.



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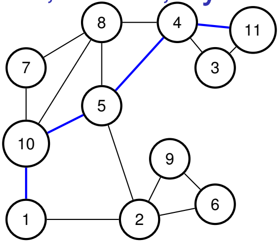
Quick Check! Length of path?

Paths, walks, cycles, tour.



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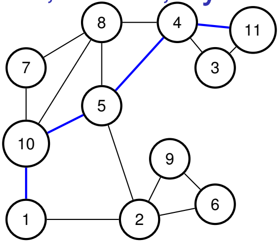
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



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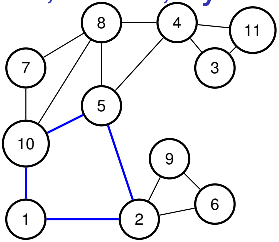
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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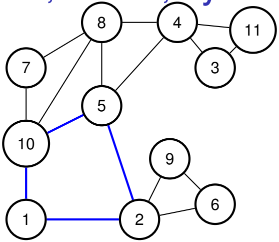
Paths, walks, cycles, tour.



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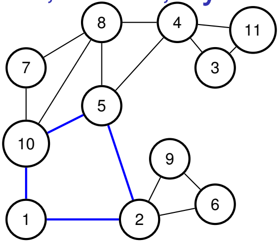
Paths, walks, cycles, tour.



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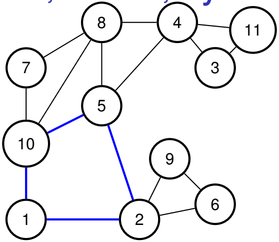
Paths, walks, cycles, tour.



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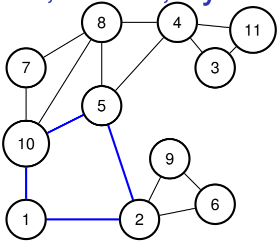
Paths, walks, cycles, tour.



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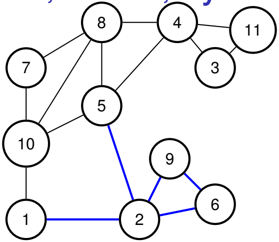

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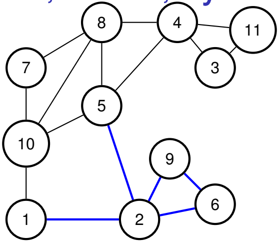
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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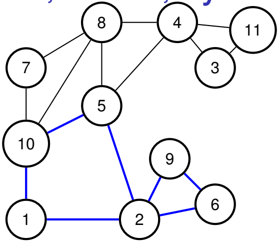

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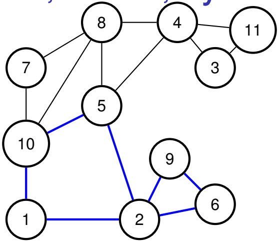
Paths, walks, cycles, tour.



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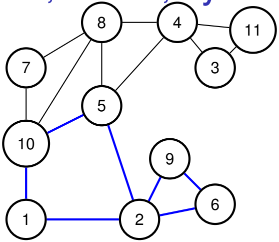

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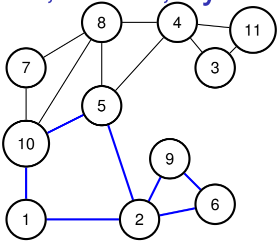

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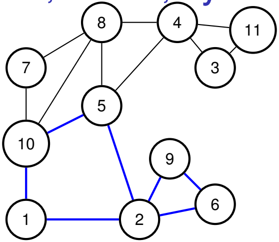
Quick Check!

Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



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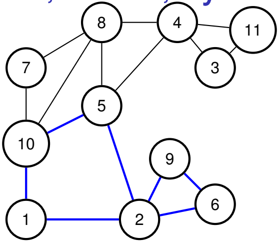

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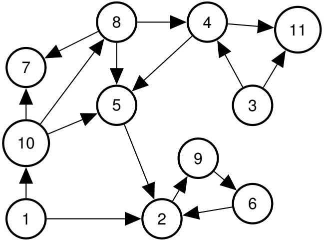
Directed Paths.



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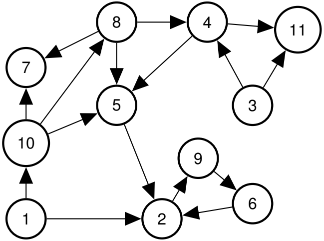

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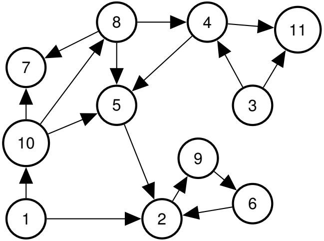

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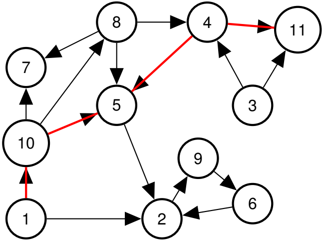

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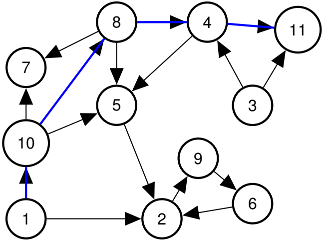

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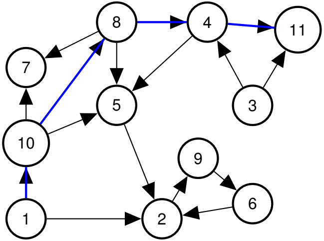

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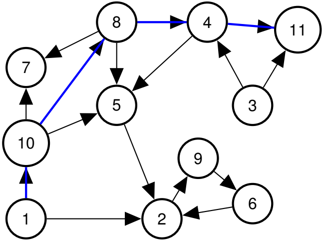

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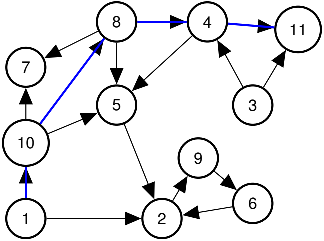

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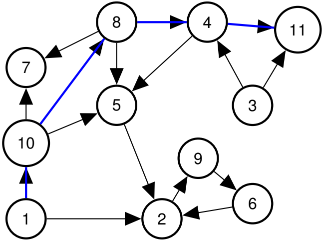

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Paths, walks, cycles, tours

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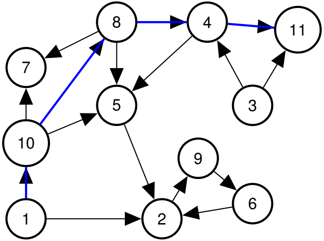

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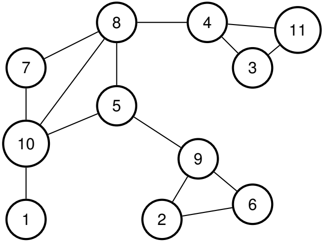
Paths, walks, cycles, tours ... are analagous to undirected now.

Connectivity: undirected graph.



u and *v* are connected if there is a path between *u* and *v*.

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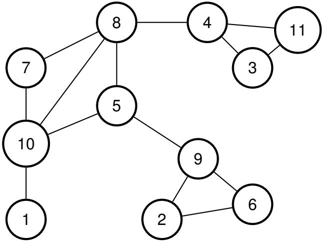

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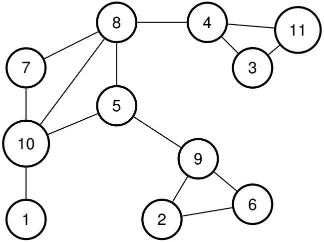

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If one vertex x is connected to every other vertex.

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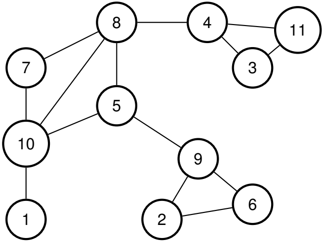

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Is graph connected?

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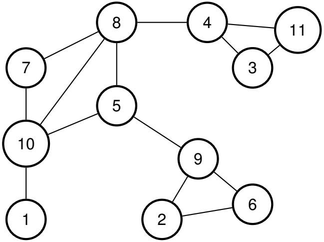

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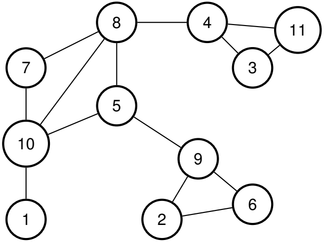

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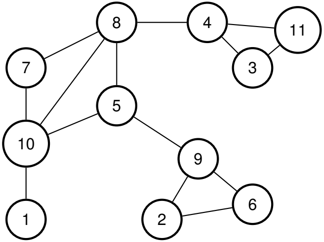

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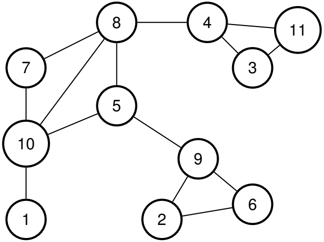

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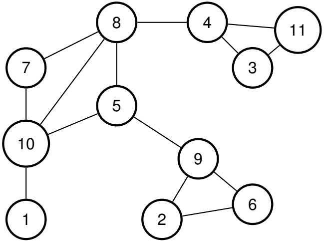

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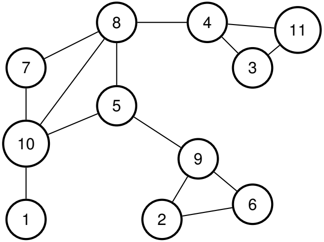

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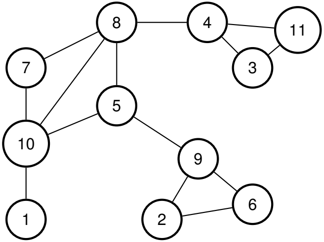

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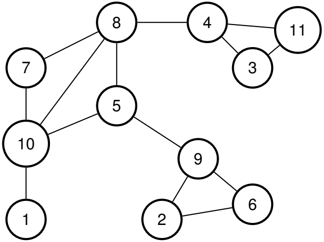

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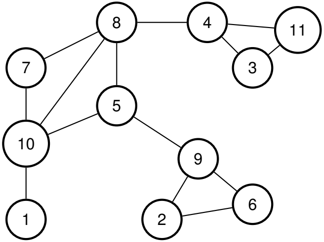

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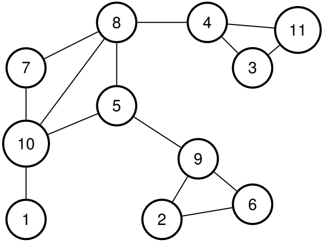

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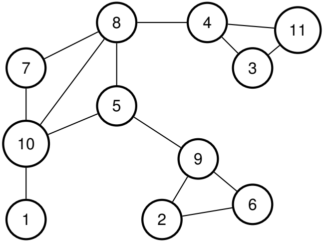
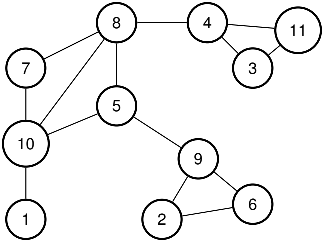
Connected Components: Quiz.



Is graph above connected?

Connected Components: Quiz.



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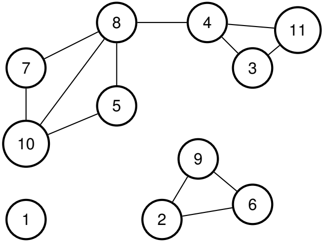

Is graph above connected? Yes!

How about now?

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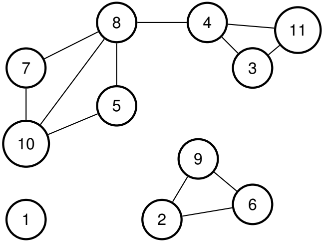

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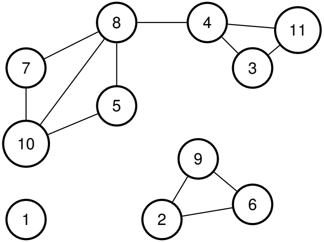

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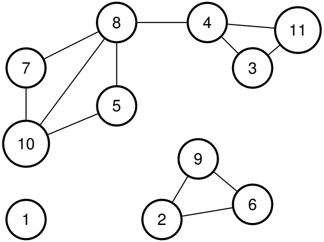
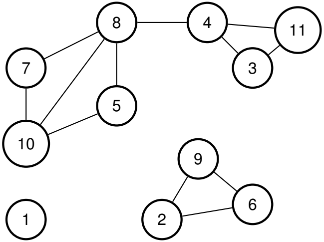


Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\},\{10,7,5,8,4,3,11\},\{2,9,6\}$.

Connected Components: Quiz.



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Connected component - maximal set of connected vertices.

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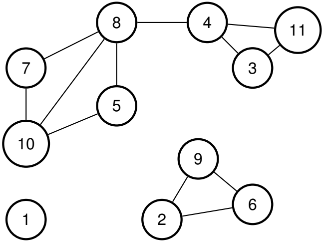

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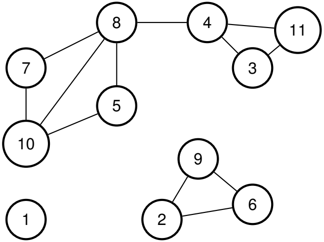
Quick Check: Is $\{10, 7, 5\}$ a connected component?

Connected Components: Quiz.



Is graph above connected? Yes!

How about now? No!

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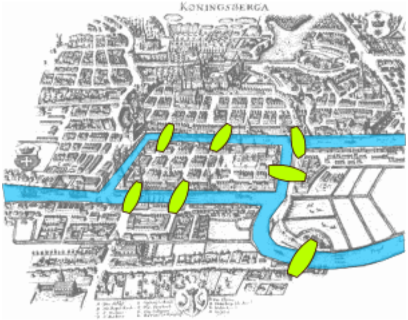
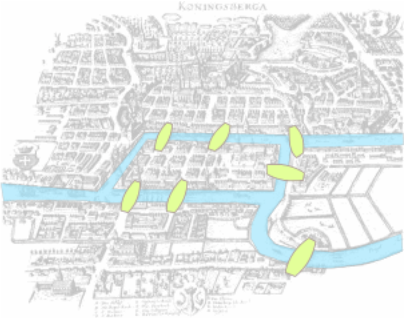
Quick Check: Is $\{10, 7, 5\}$ a connected component? No.

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?





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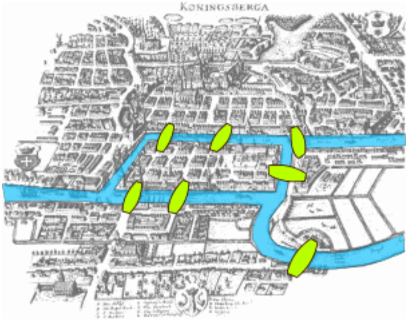

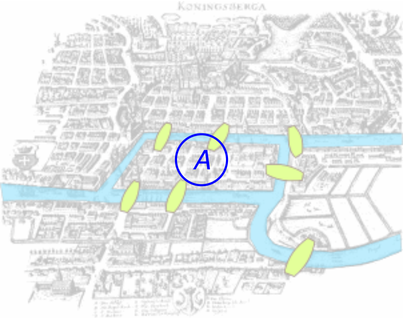

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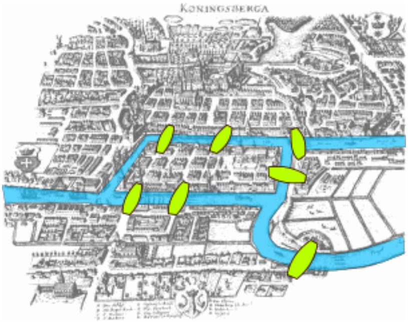

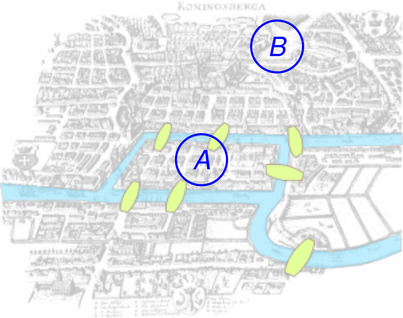

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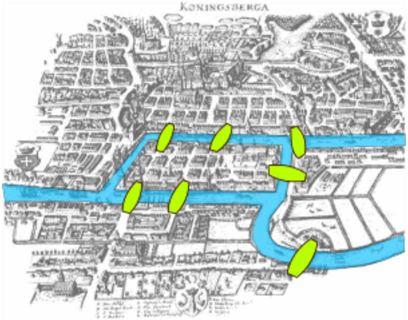

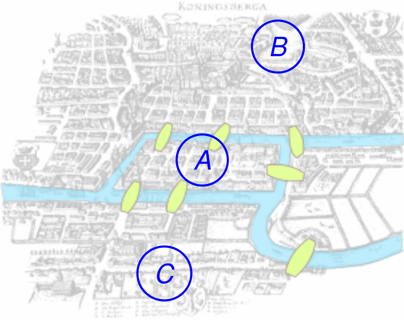

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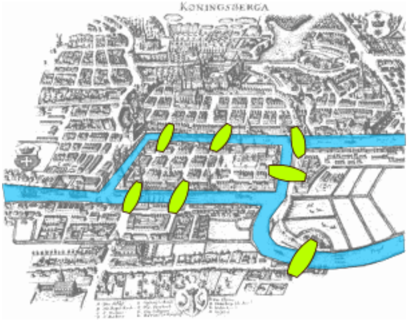

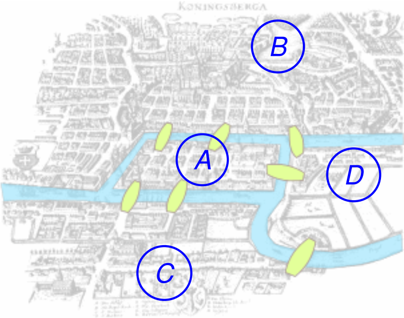

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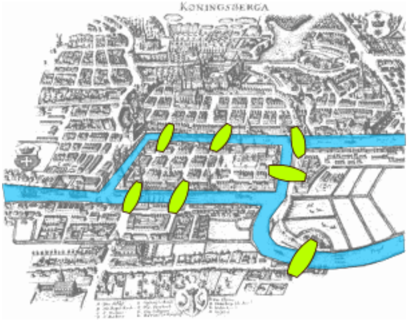

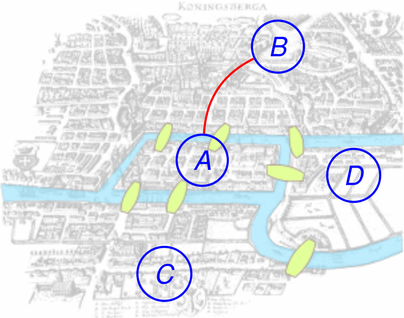

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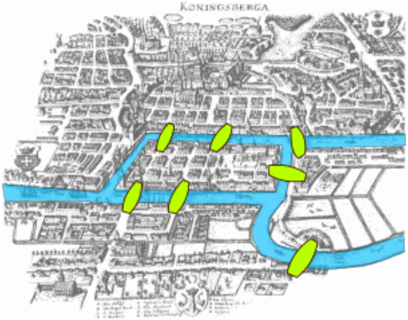

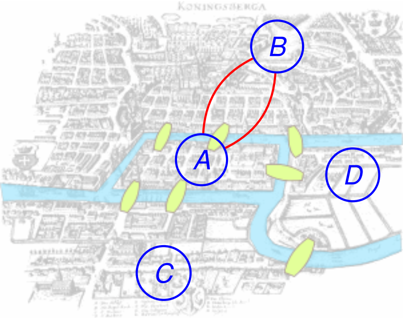

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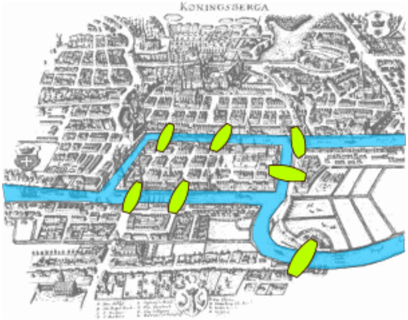

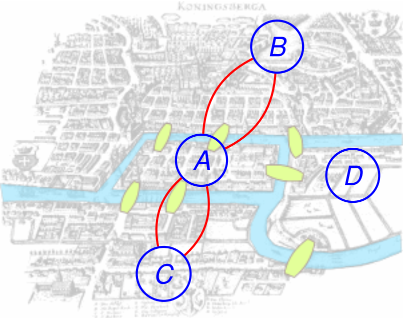

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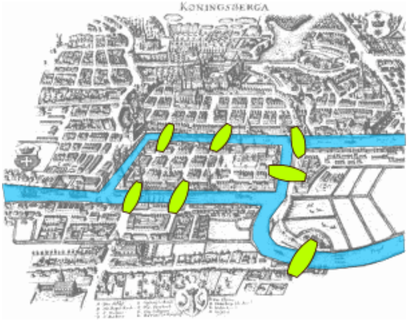

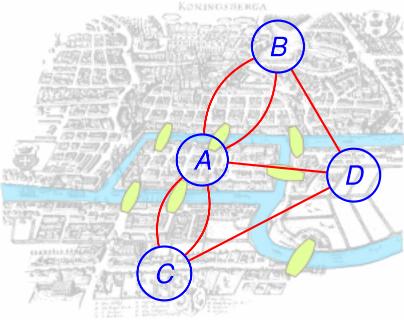

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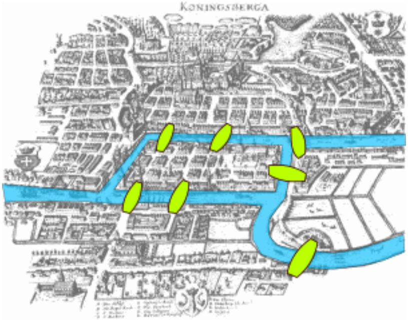

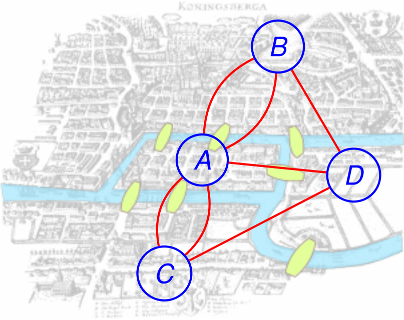

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

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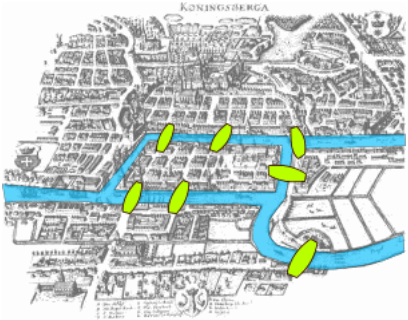

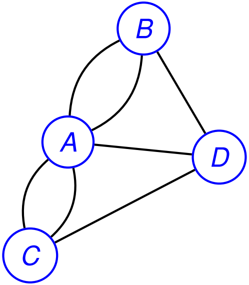

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Konigsberg bridges problem.

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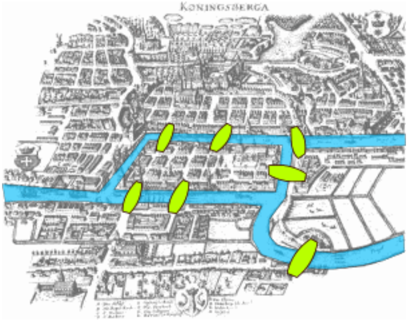

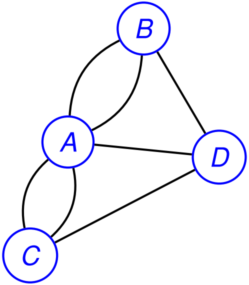

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Yes? No?

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Can you make a tour visiting each bridge exactly once?

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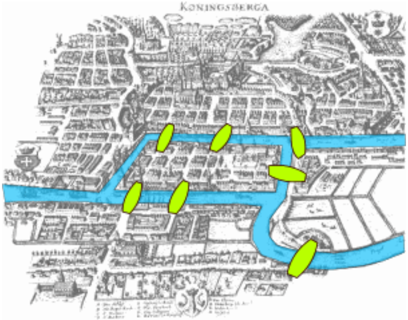

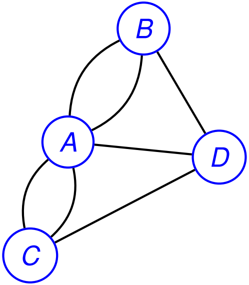

Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

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Tour enters and leaves vertex v on each visit.

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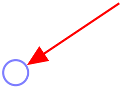

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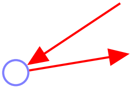

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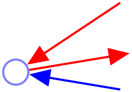

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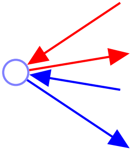

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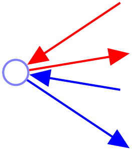

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For starting node,

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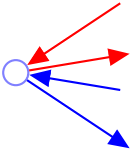

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For starting node, tour leaves first

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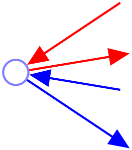

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.

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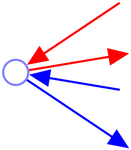

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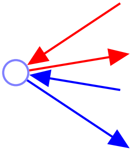

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.
Not The Hotel California.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

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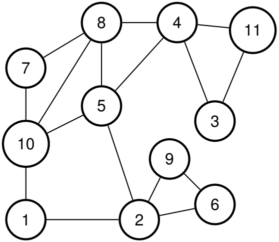

1. Take a walk starting from v (1) on "unused" edges

Finding a tour!

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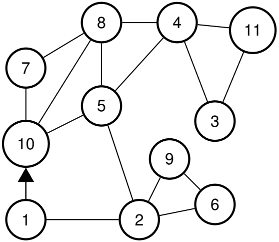

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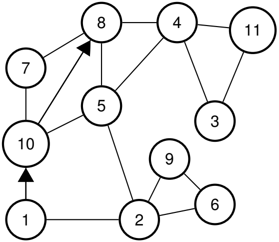

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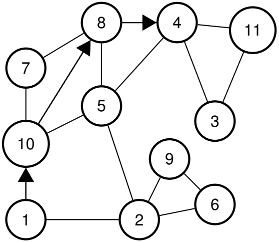

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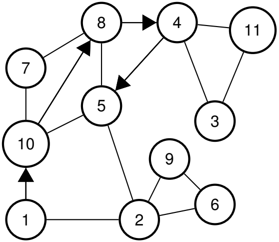

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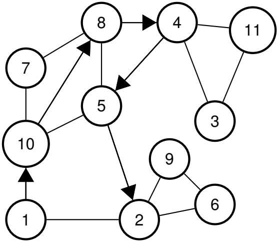

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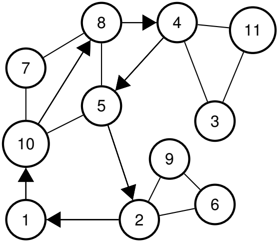

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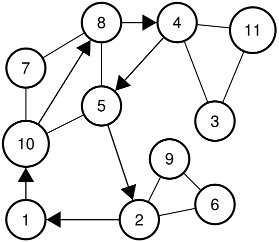

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2. Remove tour, C.

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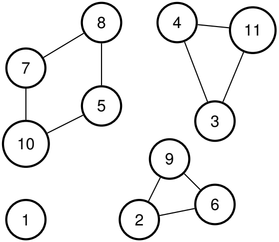

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3. Let $G_1, \ldots, G_k$ be connected components.

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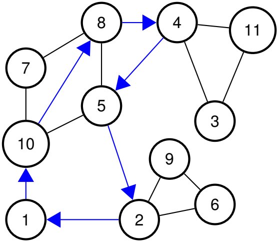

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   Each is touched by C.

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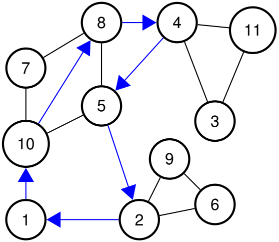

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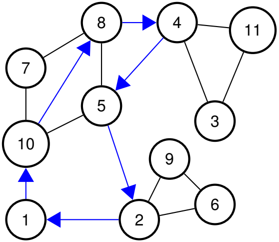

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Why? G was connected.

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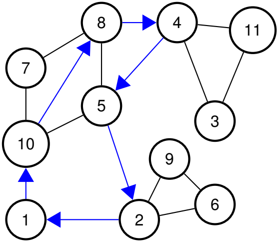

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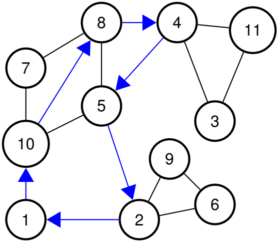

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    Example: $v_1 = 1$,

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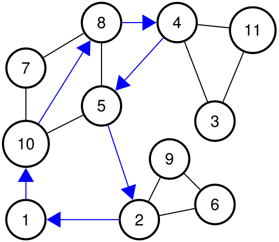

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   Example: $v_1 = 1$, $v_2 = 10$,

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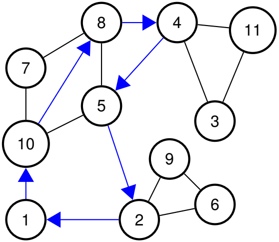

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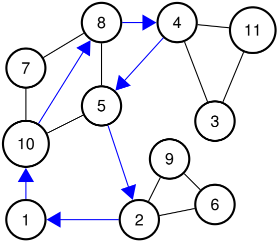

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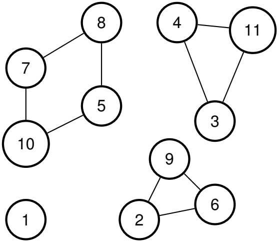

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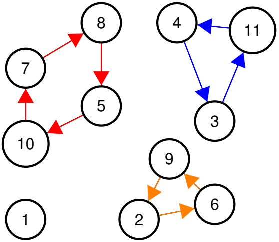

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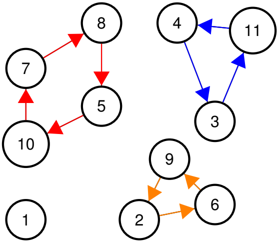

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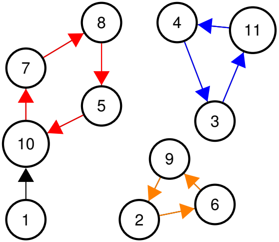

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   1,10

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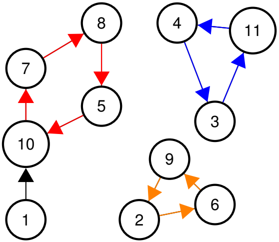

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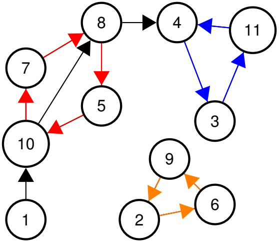
   Let v_i be (first) node in G_i touched by C.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from v_i
5. Splice together.
   1,10,7,8,5,10

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.



1. Take a walk starting from v (1) on "unused" edges
... till you get back to v.
2. Remove tour, C.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.
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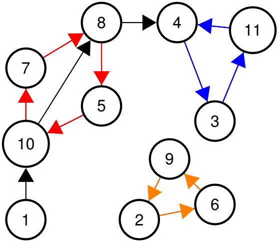

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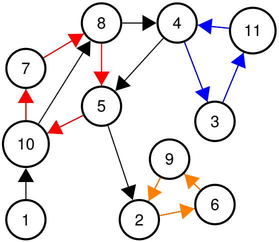

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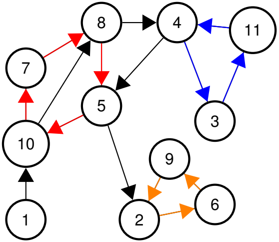

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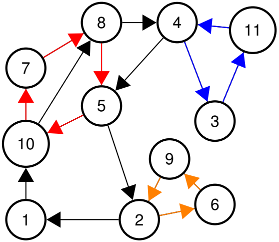

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Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Recursive/Inductive Algorithm.

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Claim: Do get back to v!

Recursive/Inductive Algorithm.

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Claim: Do get back to v!
Proof of Claim: Even degree.

Recursive/Inductive Algorithm.

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Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave

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Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
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2. Remove cycle, C, from G.

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Resulting graph may be disconnected. (Removed edges!)

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Why is there a v_i in C?

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Claim: Each vertex in each G_i has even degree and is connected.

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Prf: Tour C has even incidences to any vertex v.

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3. Find tour T_i of G_i starting/ending at v_i. Induction.

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Visits every edge once:
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  By induction for all edges in each G_i.

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Poll: Euler concepts.

Mark correct statements for a connected graph where all vertices have even degree. (Below, tours uses edges at most once, but may involve a vertex several times.

Poll: Euler concepts.

Mark correct statements for a connected graph where all vertices have even degree. (Below, tours uses edges at most once, but may involve a vertex several times.

(A) Removing a tour leaves a graph of even degree.
(B) A tour connecting a set of connected components, each with a Eulerian tour is really cool! Eulerian even.
(C) There is no hotel california in this graph.
(D) After removing a set of edges E' in a connected graph, every connected component is incident to an edge in E'
(E) If one walks on new edges, starting at v, one must eventually get back to v.
(F) Removing a tour leaves a connected graph.

Poll: Euler concepts.

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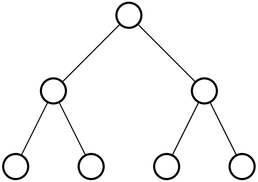
(F) Removing a tour leaves a connected graph.

Only (F) is false.

A Tree, a tree.

Graph $G = (V, E)$.

Binary Tree!



More generally.

Trees.

Definitions:

Trees.

Definitions:

A connected graph without a cycle.

Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V|-1$ edges.

Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
A connected graph where any edge removal disconnects it.

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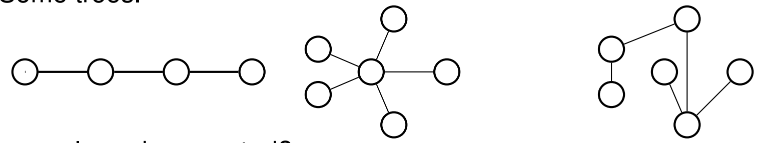
Some trees.



no cycle and connected?

Trees.

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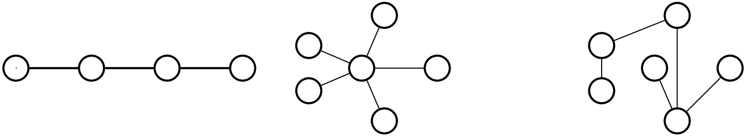

no cycle and connected? Yes.

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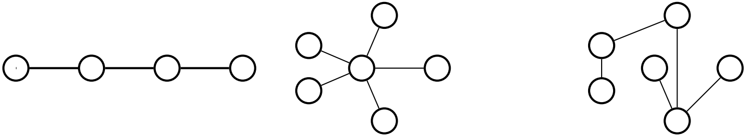

no cycle and connected? Yes.
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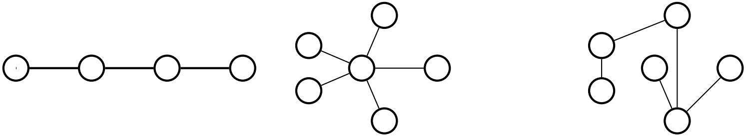

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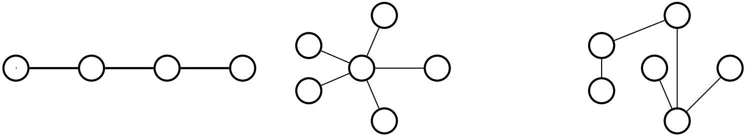

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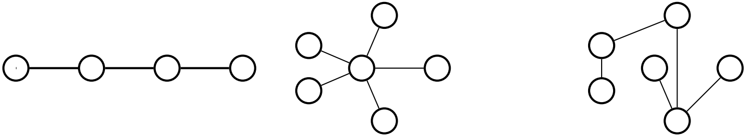

no cycle and connected? Yes.
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removing any edge disconnects it. Harder to check.

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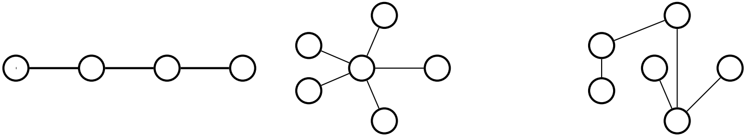

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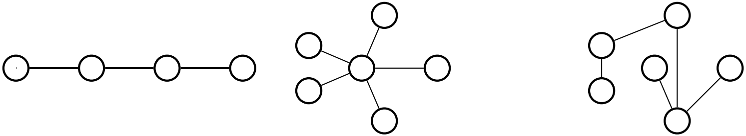
Some trees.



no cycle and connected? Yes.
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Adding any edge creates cycle.

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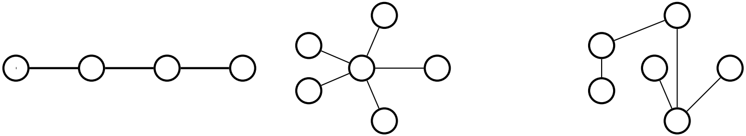

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removing any edge disconnects it. Harder to check. but yes.
Adding any edge creates cycle. Harder to check.

Trees.

Definitions:

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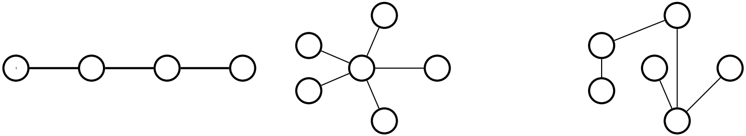
Some trees.



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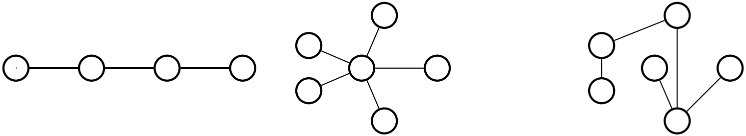

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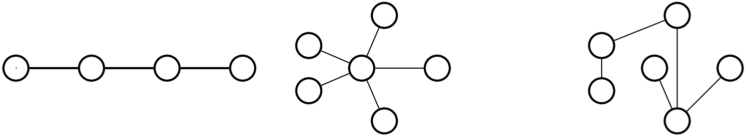

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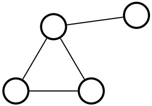

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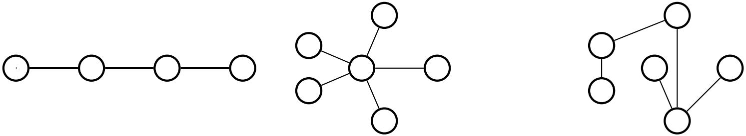

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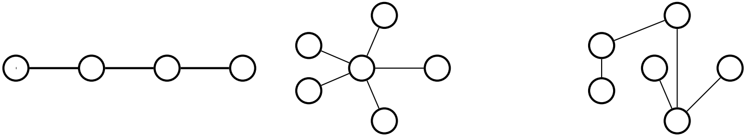

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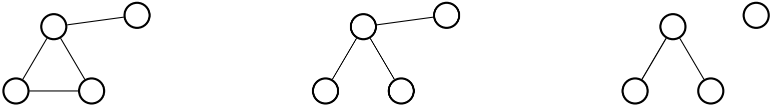

Equivalence of Definitions.

Theorem:
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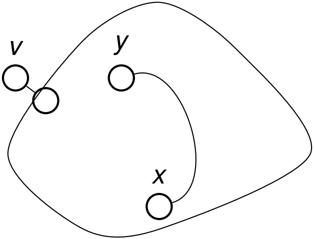

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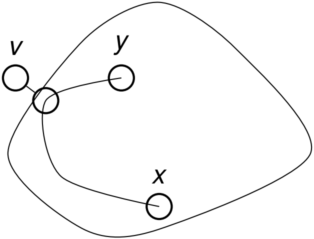

Proof of only if.

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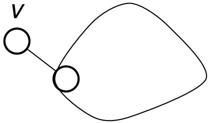

Proof of $\implies$:

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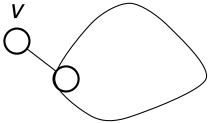
Proof of $\Longrightarrow$ **:** By induction on $|V|$.



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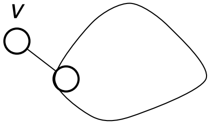

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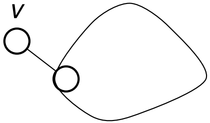

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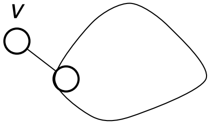

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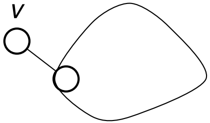

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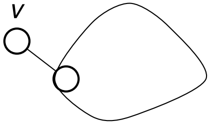

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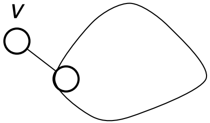

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Sum of degrees is $2|E|=2(|V|-1)=2|V|-2$

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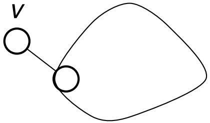

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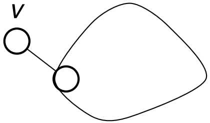

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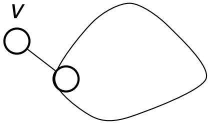

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Cuz not everyone is bigger than average!

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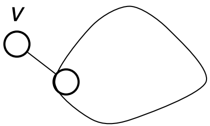

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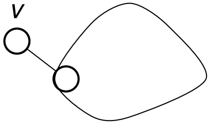

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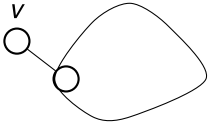

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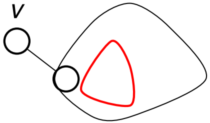

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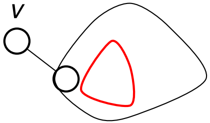

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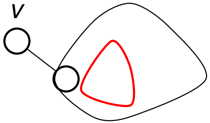

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Proof:
Walk from a vertex using untraversed edges.

Proof of if

Thm:
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Until get stuck.

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Proof of if

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Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.

Proof of if

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Until get stuck.
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Proof of Claim:
Can't visit more than once since no cycle.
Entered.

Proof of if

Thm:
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Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave.

Proof of if

Thm:
"G is connected and has no cycles"
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Entered. Didn't leave. Only one incident edge.

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Removing node doesn't create cycle.

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New graph is connected.

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Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.

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By induction $G - v$ has $|V| - 2$ edges.

Proof of if

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Removing node doesn't create cycle.
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By induction $G-v$ has $|V|-2$ edges.
G has one more or $|V|-1$ edges.

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G has one more or $|V|-1$ edges. $\square$

Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V| - 1$ edges.

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(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2-2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.

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(B), (C), (D) are true

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G: n vertices and $n-1$ edges and connected.
remove degree 1 vertex.
$n-1$ vertices, $n-2$ edges and connected $\implies$ acyclic.
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degree 1 vertex is not in a cycle.
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