CS70: New Discussion Format

Small group:

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Small group:

Three modes of working.

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Three modes of working.
(A) Individual working.

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Three modes of working.

(A) Individual working.

(B) Pairs working together.

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(C) Pairs: one works/one forces talking.

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(1) Experience. (years and years, faculty agree.)

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We would like you to be "happy"

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Our job is to have you learn.
We would like you to be “happy” in the moment.
But the result is what is important.

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But the result is what is important.

Be nice to the TA's.

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But the result is what is important.

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CS70: Lecture 9. Outline.

1. Public Key Cryptography
2. RSA system
   2.1 Efficiency: Repeated Squaring.
   2.2 Correctness: Fermat's Theorem.
   2.3 Construction.
3. Warnings.

Simple Chinese Remainder Theorem.

Simple Chinese Remainder Theorem.

My love is won.

Simple Chinese Remainder Theorem.

My love is won. Zero and One.

Simple Chinese Remainder Theorem.

My love is won. Zero and One. Nothing and nothing done.

Simple Chinese Remainder Theorem.

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Find $x = a \pmod{m}$ and $x = b \pmod{n}$

Simple Chinese Remainder Theorem.

My love is won. Zero and One. Nothing and nothing done.

Find $x = a \pmod{m}$ and $x = b \pmod{n}$ where gcd(m,n)=1.

Simple Chinese Remainder Theorem.

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Find $x = a \pmod{m}$ and $x = b \pmod{n}$ where gcd(m,n)=1.

CRT Thm: There is a unique solution $x \pmod{mn}$.

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Find $x = a \pmod{m}$ and $x = b \pmod{n}$ where gcd(m, n)=1.

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Proof (solution exists):

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Proof (solution exists):
Consider $u = n(n^{-1} \pmod{m})$.

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Find $x = a \pmod{m}$ and $x = b \pmod{n}$ where gcd(m,n)=1.

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$u = 0 \pmod{n}$

Simple Chinese Remainder Theorem.

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Consider $v = m(m^{-1} \pmod{n})$.

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Consider $v = m(m^{-1} \pmod{n})$.
$v = 1 \pmod{n}$ $\qquad$ $v = 0 \pmod{m}$
Let $x = au + bv$.

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Consider $v = m(m^{-1} \pmod{n})$.
$v = 1 \pmod{n}$ $\quad v = 0 \pmod{m}$
Let $x = au + bv$.
$x = a \pmod{m}$ since $bv = 0 \pmod{m}$ and $au = a \pmod{m}$

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Find $x = a \pmod{m}$ and $x = b \pmod{n}$ where gcd(m,n)=1.

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Let $x = au + bv$.
$x = a \pmod{m}$ since $bv = 0 \pmod{m}$ and $au = a \pmod{m}$
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Consider $v = m(m^{-1} \pmod{n})$.
$v = 1 \pmod{n}$ $\qquad v = 0 \pmod{m}$
Let $x = au + bv$.
$x = a \pmod{m}$ since $bv = 0 \pmod{m}$ and $au = a \pmod{m}$
$x = b \pmod{n}$ since $au = 0 \pmod{n}$ and $bv = b \pmod{n}$
This shows there is a solution. $\square$

Simple Chinese Remainder Theorem.

CRT Thm: There is a unique solution $x \pmod{mn}$.

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Proof (uniqueness):

Simple Chinese Remainder Theorem.

CRT Thm: There is a unique solution $x \pmod{mn}$.

Proof (uniqueness):
If not, two solutions, x and y.

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If not, two solutions, x and y.

Simple Chinese Remainder Theorem.

CRT Thm: There is a unique solution $x \pmod{mn}$.

Proof (uniqueness):
If not, two solutions, x and y.

$(x-y) \equiv 0 \pmod{m}$ and $(x-y) \equiv 0 \pmod{n}$.

Simple Chinese Remainder Theorem.

CRT Thm: There is a unique solution $x \pmod{mn}$.

Proof (uniqueness):
If not, two solutions, x and y.

$(x-y) \equiv 0 \pmod{m}$ and $(x-y) \equiv 0 \pmod{n}$.
$\implies (x-y)$ is multiple of m and n

Simple Chinese Remainder Theorem.

CRT Thm: There is a unique solution $x \pmod{mn}$.

Proof (uniqueness):
If not, two solutions, x and y.

$(x-y) \equiv 0 \pmod{m}$ and $(x-y) \equiv 0 \pmod{n}$.
$\implies (x-y)$ is multiple of m and n
$\gcd(m,n) = 1 \implies$ no common primes in factorization m and n

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CRT Thm: There is a unique solution $x \pmod{mn}$.

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If not, two solutions, x and y.

$(x-y) \equiv 0 \pmod{m}$ and $(x-y) \equiv 0 \pmod{n}$.
$\implies (x-y)$ is multiple of m and n
$\gcd(m,n) = 1 \implies$ no common primes in factorization m and n
$\implies mn|(x-y)$

Simple Chinese Remainder Theorem.

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Proof (uniqueness):
If not, two solutions, x and y.

$(x-y) \equiv 0 \pmod{m}$ and $(x-y) \equiv 0 \pmod{n}$.
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$\implies mn|(x-y)$
$\implies x-y \geq mn$

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$\implies mn|(x-y)$
$\implies x-y \geq mn \implies x,y \notin \{0,\dots,mn-1\}$.

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$(x-y) \equiv 0 \pmod{m}$ and $(x-y) \equiv 0 \pmod{n}$.
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$\implies mn|(x-y)$
$\implies x - y \geq mn \implies x, y \notin \{0, \ldots, mn-1\}$.
Thus, only one solution modulo mn.

Simple Chinese Remainder Theorem.

CRT Thm: There is a unique solution $x \pmod{mn}$.

Proof (uniqueness):
If not, two solutions, x and y.

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Thus, only one solution modulo mn. $\square$

Isomorphisms.

Bijection:

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$f(x) = ax \pmod{m}$ if $gcd(a, m) = 1$.

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Simplified Chinese Remainder Theorem:

Isomorphisms.

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$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

Simplified Chinese Remainder Theorem:

If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where

$x = a \pmod{m}$ and $x = b \pmod{n}$.

Isomorphisms.

Bijection:

$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

Simplified Chinese Remainder Theorem:
If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Isomorphisms.

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Simplified Chinese Remainder Theorem:

If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
$x = a \pmod{m}$ and $x = b \pmod{n}$.

Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Isomorphisms.

Bijection:

$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

Simplified Chinese Remainder Theorem:
If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a', b') = (2,4)$, then $x = 22 \pmod{45}$.

Isomorphisms.

Bijection:

$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

Simplified Chinese Remainder Theorem:

If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a', b') = (2,4)$, then $x = 22 \pmod{45}$.

Now consider:

Isomorphisms.

Bijection:

$f(x) = ax \pmod{m}$ if $gcd(a, m) = 1$.

Simplified Chinese Remainder Theorem:
If $gcd(n, m) = 1$, there is unique $x \pmod{mn}$ where
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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a, b) = (3, 7)$ then $x = 43 \pmod{45}$.

Consider $(a', b') = (2, 4)$, then $x = 22 \pmod{45}$.

Now consider: $(a, b) + (a', b') = (0, 2)$.

Isomorphisms.

Bijection:

$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

Simplified Chinese Remainder Theorem:
If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
$x = a \pmod{m}$ and $x = b \pmod{n}$.

Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a',b') = (2,4)$, then $x = 22 \pmod{45}$.

Now consider: $(a,b) + (a',b') = (0,2)$.

What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Isomorphisms.

Bijection:

$f(x) = ax \pmod{m}$ if $gcd(a, m) = 1$.

Simplified Chinese Remainder Theorem:

If $gcd(n, m) = 1$, there is unique $x \pmod{mn}$ where
$x = a \pmod{m}$ and $x = b \pmod{n}$.

Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a, b) = (3, 7)$ then $x = 43 \pmod{45}$.

Consider $(a', b') = (2, 4)$, then $x = 22 \pmod{45}$.

Now consider: $(a, b) + (a', b') = (0, 2)$.

What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65$

Isomorphisms.

Bijection:

$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

Simplified Chinese Remainder Theorem:
If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
$x = a \pmod{m}$ and $x = b \pmod{n}$.

Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a',b') = (2,4)$, then $x = 22 \pmod{45}$.

Now consider: $(a,b) + (a',b') = (0,2)$.

What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65 = 20 \pmod{45}$.

Isomorphisms.

Bijection:

$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

Simplified Chinese Remainder Theorem:
If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
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the actions under $\pmod{5}$, $\pmod{9}$

correspond to actions in $\pmod{45}$!

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
$y = 4 \mod 7$ **and** $y = 3 \mod 6$

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What's true?

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What's true?

(A) $x + y = 2 \mod 7$
(B) $x + y = 2 \mod 6$
(C) $xy = 3 \mod 6$
(D) $xy = 6 \mod 7$
(E) $x = 5 \mod 42$
(F) $y = 39 \mod 42$

Poll

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What's true?

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All true.

Xor

Computer Science:

Xor

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1 - True
0 - False

Xor

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$1 \vee 1 = 1$

Xor

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$A \oplus B$ - Exclusive or.

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Note: Also modular addition modulo 2!

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Property: $A \oplus B \oplus B = A$.

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Cryptography ...

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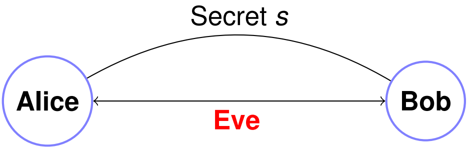
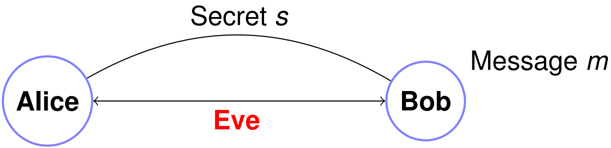


Cryptography ...



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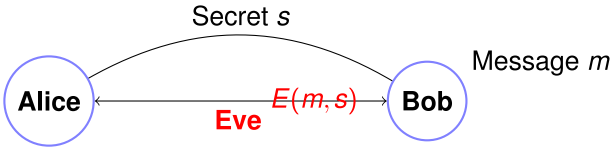

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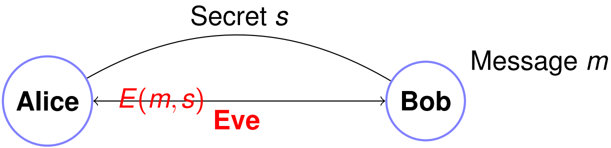

Cryptography ...

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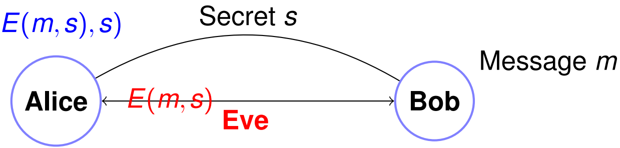

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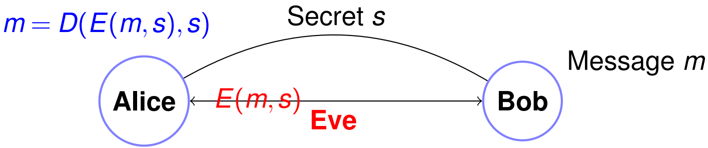
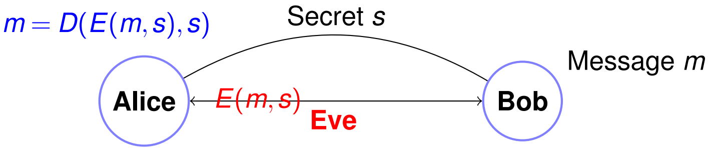


Example:

Cryptography ...



Example:
One-time Pad: secret s is string of length $|m|$.

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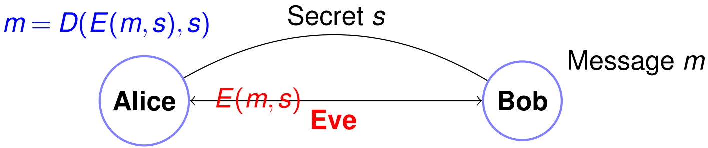

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$m = 1010101110101101$

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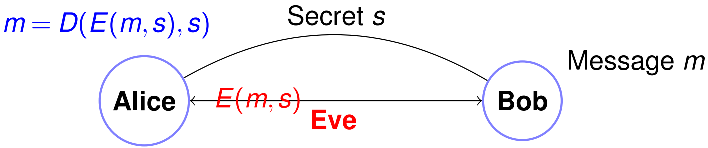

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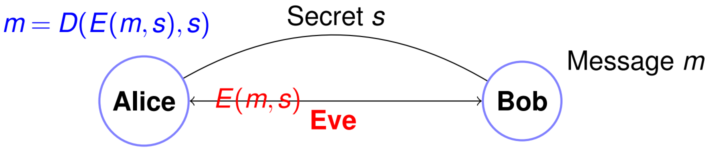

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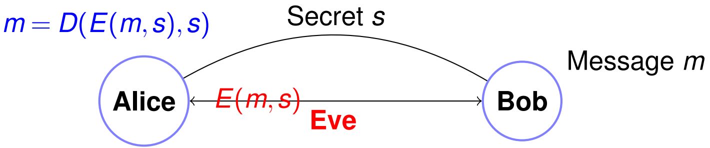
$E(m,s)$ – bitwise $m \oplus s$.

Cryptography ...



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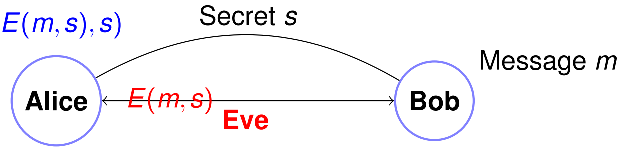

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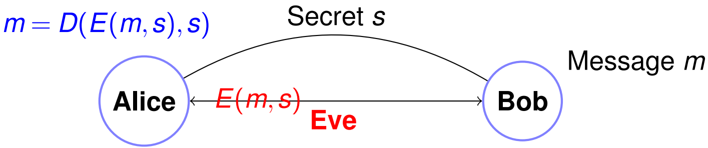

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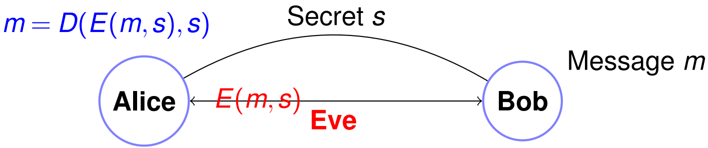

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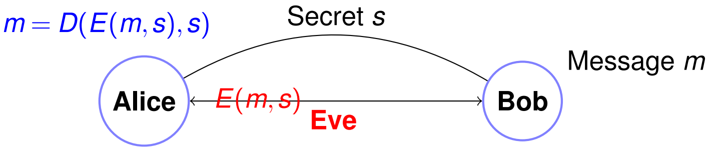

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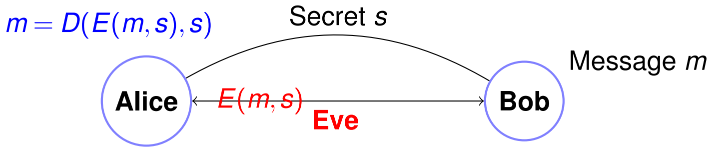
Cryptography ...



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Disadvantages:

Shared secret!

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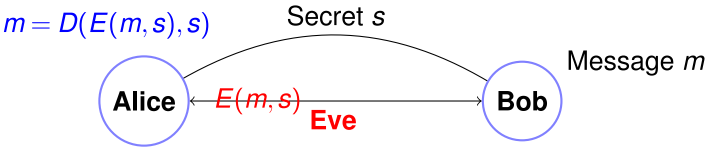

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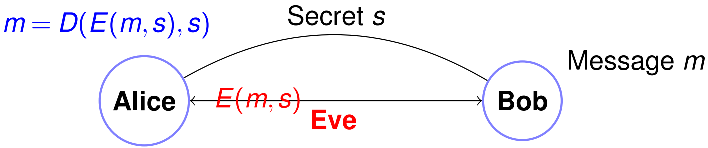

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Disadvantages:

Shared secret!

Uses up one time pad..or less and less secure.

Public key crypography.

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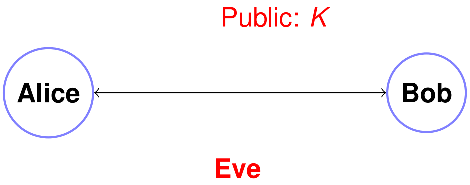

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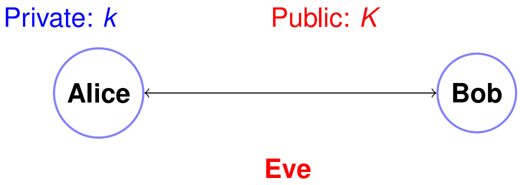

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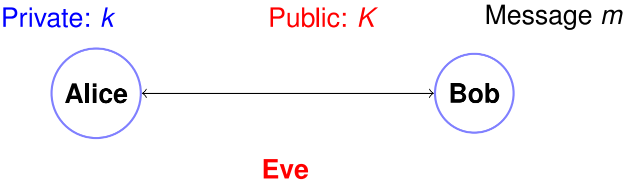

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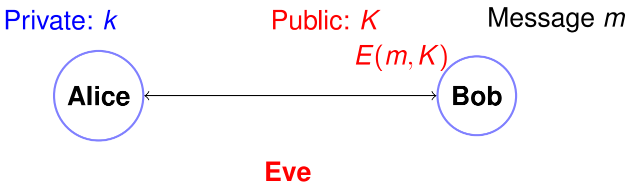

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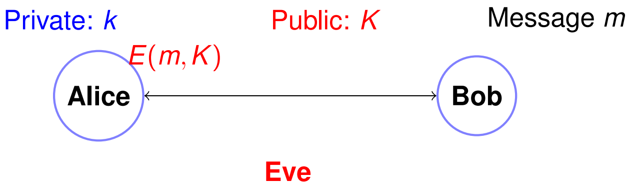
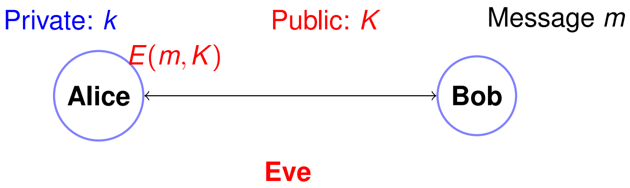


Public key crypography.

$m = D(E(m,K),k)$



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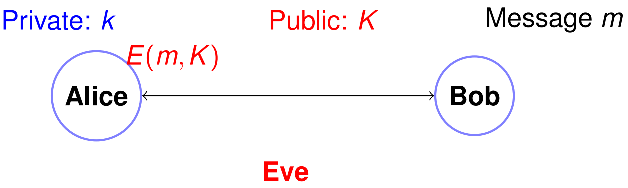
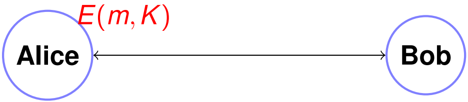


Everyone knows key K!

Public key crypography.

$m = D(E(m, K), k)$

Private: k Public: K Message m



Eve

Everyone knows key K!
Bob (and Eve

Public key crypography.

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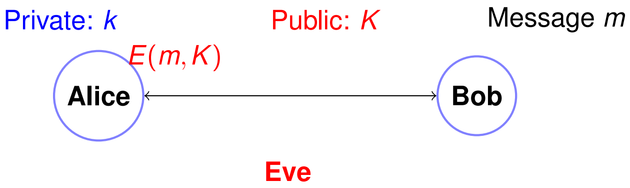

Everyone knows key K!
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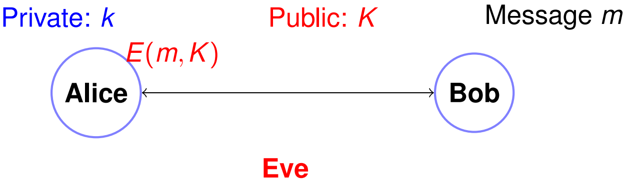

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Public key crypography.

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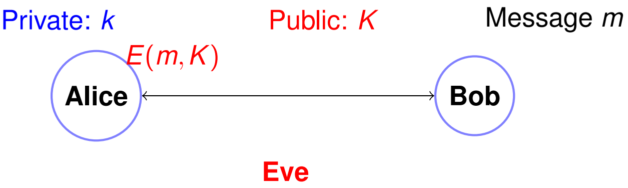

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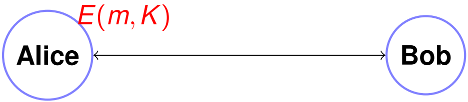
Private: k Public: K Message m



Eve

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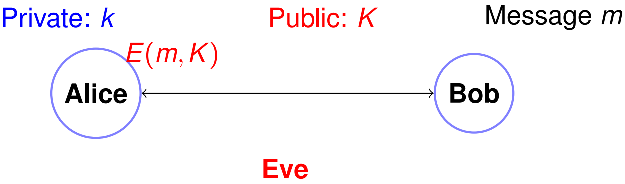

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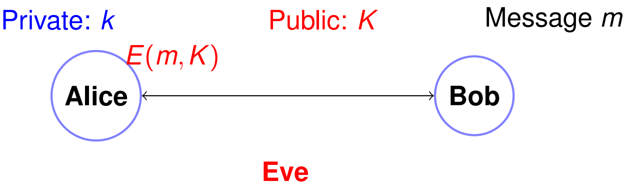

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Is this even possible?

Is public key crypto possible?

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Iterative Extended GCD.

Example: $p = 7$, $q = 11$.

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$d = e^{-1} = -17 = 43 = \pmod{60}$

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In general, $O(N)$ or $O(2^n)$ multiplications!

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$51^{32} \cdot 51^{8} \cdot 51^{2} \cdot 51^{1} = (60)*(53)*(60)*(51) \equiv 2 \pmod{77}$.

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$51^{16} = (51^{8})*(51^{8}) = 53*53 = 2809 \equiv 37 \pmod{77}$
$51^{32} = (51^{16})*(51^{16}) = 37*37 = 1369 \equiv 60 \pmod{77}$

5 more multiplications.

$51^{32} \cdot 51^{8} \cdot 51^{2} \cdot 51^{1} = (60)*(53)*(60)*(51) \equiv 2 \pmod{77}$.

Decoding got the message back!

Repeated squaring.

Notice: $43 = 32+8+2+1$ or 101011 in binary.

$51^{43} = 51^{32+8+2+1} = 51^{32} \cdot 51^{8} \cdot 51^{2} \cdot 51^{1} \pmod{77}$.

3 multiplications sort of...

Need to compute $51^{32} \ldots 51^{1}$.?

$51^{1} \equiv 51 \pmod{77}$

$51^{2} = (51)*(51) = 2601 \equiv 60 \pmod{77}$

$51^{4} = (51^{2})*(51^{2}) = 60*60 = 3600 \equiv 58 \pmod{77}$

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Repeated Squaring took 8 multiplications

Repeated squaring.

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3 multiplications sort of...
Need to compute $51^{32} \ldots 51^{1}$.?
$51^1 \equiv 51 \pmod{77}$
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5 more multiplications.

$51^{32} \cdot 51^{8} \cdot 51^{2} \cdot 51^{1} = (60)*(53)*(60)*(51) \equiv 2 \pmod{77}$.

Decoding got the message back!

Repeated Squaring took 8 multiplications versus 42.

Repeated Squaring: x^y

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute x^1,

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2,$

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2, x^4,$

Repeated Squaring: x^y

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Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2, x^4, \ldots, x^{2^{\lfloor \log y \rfloor}}$.

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2, x^4, \ldots, x^{2^{\lfloor \log y \rfloor}}$.
2. Multiply together x^i where the $(\log(i))$th bit of y (in binary) is 1.

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

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   Example:

Repeated Squaring: x^y

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   Example: $43 = 101011$ in binary.

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   Example: $43 = 101011$ in binary.
   $$x^{43} = x^{32} * x^8 * x^2 * x^1.$$

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Modular Exponentiation: $x^y \mod N$.

Repeated Squaring: x^y

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Modular Exponentiation: $x^y \mod N$. All n-bit numbers. Repeated Squaring:

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Modular Exponentiation: $x^y \mod N$. All n-bit numbers. Repeated Squaring:
$O(n)$ multiplications.

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Modular Exponentiation: $x^y \mod N$. All n-bit numbers. Repeated Squaring:

$O(n)$ multiplications.
$O(n^2)$ time per multiplication.

Repeated Squaring: x^y

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Modular Exponentiation: $x^y \mod N$. All n-bit numbers. Repeated Squaring:

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Conclusion: $x^y \mod N$

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

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Modular Exponentiation: $x^y \mod N$. All n-bit numbers. Repeated Squaring:

$O(n)$ multiplications.
$O(n^2)$ time per multiplication.
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Conclusion: $x^y \mod N$ takes $O(n^3)$ time.

RSA is pretty fast.

Modular Exponentiation: $x^y \mod N$.

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$E(m,(N,e)) = m^e \pmod{N}$.

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Decoding.

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Always decode correctly?

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versus $\quad a^{k(p-1)(q-1)+1} = a \pmod{pq}$.

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versus $\quad a^{k(p-1)(q-1)+1} = a \pmod{pq}$.

Similar, not same, but useful.

Correct decoding...

Fermat's Little Theorem: For prime p, and $a \not\equiv 0 \pmod{p}$,

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Each of $2, \ldots (p-1)$ has an inverse modulo p,

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□

Poll

Mark what is true.

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(A) $2^7 = 1 \mod 7$
(B) $2^6 = 1 \mod 7$
(C) $2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7$ are distinct mod 7.
(D) $2^1, 2^2, 2^3, 2^4, 2^5, 2^6$ are distinct mod 7
(E) $2^{15} = 2 \mod 7$
(F) $2^{15} = 1 \mod 7$

Poll

Mark what is true.

(A) $2^7 = 1 \mod 7$
(B) $2^6 = 1 \mod 7$
(C) $2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7$ are distinct mod 7.
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(B), (F)

Always decode correctly? (cont.)

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From CRT: $y = x \pmod{p}$ and $y = x \pmod{q} \implies y = x$.

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All steps are polynomial in $O(\log N)$, the number of bits.

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Breaking in general sense $\implies$ factoring algorithm.

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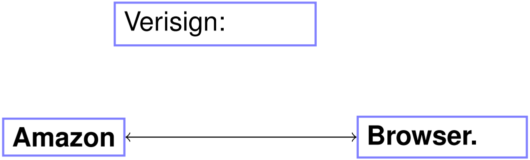
CS161...

Signatures using RSA.



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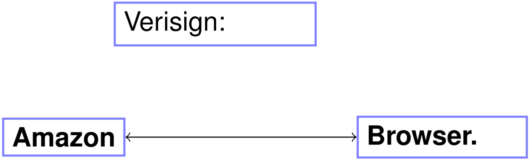

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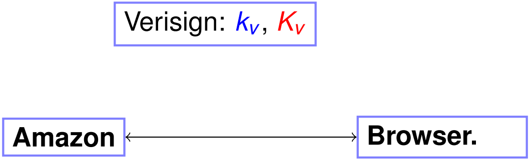

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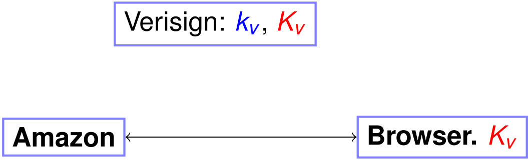

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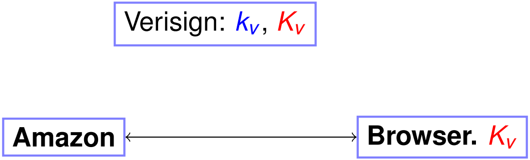

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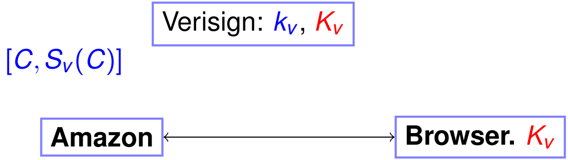

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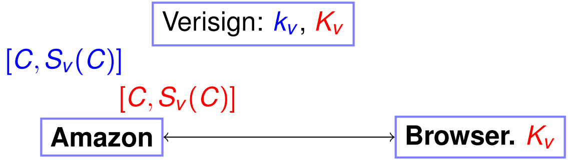

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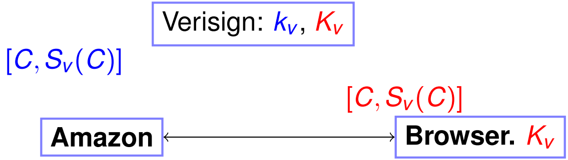

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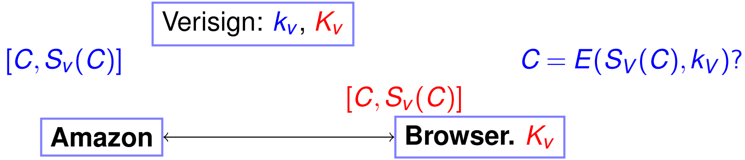

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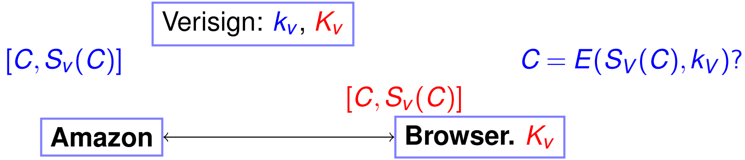

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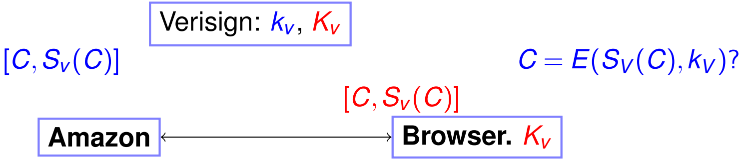

Certificate Authority: Verisign, GoDaddy, DigiNotar,...

Verisign's key: $K_V = (N, e)$ and $k_V = d$ $(N = pq.)$

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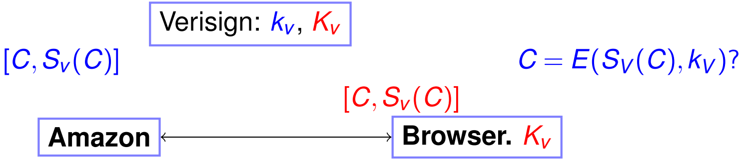

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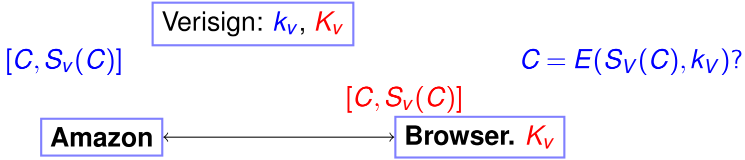

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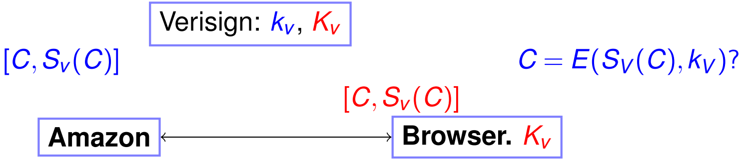

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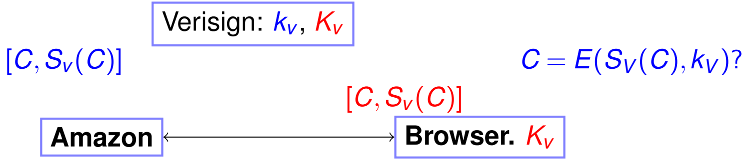

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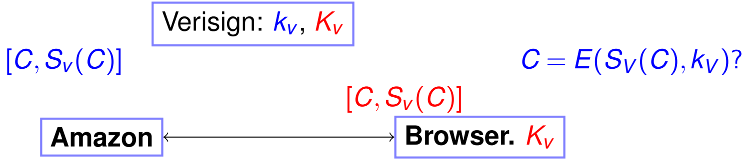

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Security: Eve can't forge unless she "breaks" RSA scheme.

RSA

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Public Key Cryptography:

RSA

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Signature authority has public key (N,e).

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(A) Given message/signature (x,y) : check $y^d = x \pmod{N}$
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RSA Scheme:

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